SOLOW'S HARROD:

TRANSFORMING CYCLICAL DYNAMICS INTO A MODEL OF LONG-RUN GROWTH

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Abstract

Modern growth theory derives mostly from Robert Solow's "A Contribution to the Theory of Economic Growth" (1956). Solow's own interpretation locates the origins of his "Contribution" in his view that the growth model of Roy Harrod implied a tendency toward progressive collapse of the economy. He formulates his view in terms of Harrod's invoking a fixed-coefficients production function. We challenge Solow's reading of Harrod's "Essay in Dynamic Theory," arguing that Harrod's object in providing a "dynamic" theory had little to do with the problem of long-run growth as Solow understood it, but instead addressed medium-run fluctuations, the "inherent instability" of economies. It was an attempt to isolate conditions under which the economy might tend to run below potential. In making this argument, Harrod does not appeal to a fixed-coefficients production function – or to any production function at all, as that term is understood by Solow. Solow interpreted Harrod's "Essay" in the light of a particular culture of understanding grounded in the practice of formal modeling that emerged in economics in the post-World War II period. The fate of Harrod's analysis is a case study in the difficulties in communicating across distinct interpretive communities and of the potential for losing content and insights in the process. From Harrod's English Keynesian point of view, Solow's interpretation arose out of a culture of misunderstanding, and his objects – particularly, of trying to account for a tendency of the economy toward chronic recessions – were lost to the mainstream literature.

Keywords: economic growth**,** Roy Harrod, Robert Solow, dynamics, dynamic instability, knife-edge, warranted rate of growth, natural rate of growth

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The beliefs of economists about the history of their own discipline often reveal a failure to have read the original sources or to have considered their meaning. They are too willing to take history as having been faithfully transmitted by other economists or through the textbooks. The textbook narratives themselves more nearly reflect the self-image of the contemporary community than any compelling account of the actual history of the field. Hence, economists are apt to take positions on history that are strong nearly in proportion to their being wrong. We were brought up as economists with such a strong view of Roy Harrod's "growth model."

In their textbook, Robert Barro and Xavier Sala-i-Martin (2003, p. 17) write that Harrod "used production functions with little substitutability among the inputs to argue that the capitalist system is inherently unstable." Nearly every textbook that mentions Harrod in the context of growth – and many ignore him altogether – takes a similar line (cf. Besomi 1999, p. 209). Indeed, it is the story that one of the authors of this article has told himself (Hoover 2003, p. 413)[1]. The common history of growth theory provides us with a narrative that presents Harrod's "Essay in Dynamic Theory" as the origin of modern growth theory. Harrod, it is said, offered a model of long-term economic growth that ignored substitution of factors of production and, therefore, possessed an unreasonable "knife-edge" property in which any step away from the warranted rate of growth led inexorably to collision with a full employment ceiling or to mass unemployment and depression. The protagonist of this narrative, Robert Solow, corrected

[1] Another recent interpretation is to be found in La Grandville (2007): "[I]n the first part of the twentieth century a fixed relationship was posited between factor inputs and output. . . In one form or another this led . . . Harrod (1948), and Domar (1946) to dire predictions about the future of the economy: it was bound either to a waste of resources or to ever-increasing unemployment, unless the growth rate of labour happened to be exactly equal to the savings rate divided by the fixed capital–output ratio" (La Grandville 2007, p. 16).

Harrod's knife edge and transformed his model into the basis of modern growth theory. The now canonical story of the development of growth theory originates with Solow himself: Both the term "knife-edge" and the proposal that Harrod's instability arises from a fixed-coefficients production function – and can be cured by a flexible-coefficients production function – go back to Solow's "Contribution to the Theory of Economic Growth" (1956). He showed that the price system would guarantee that deviations of the warranted rate of growth from the natural rate (i.e., the rate consistent with the growth of population and technology) would be self-correcting. Far from being balanced on a knife-edge, long-term growth was stable.

Reading Harrod's (1939) and Solow's (1956) articles side by side shows that this potted history is a misreading: not only was Harrod interested in very different questions than Solow, he also applied a different framework of analysis and came to different conclusions. Daniele Besomi shows how the interpretation of Harrod's dynamics as a theory of growth originated in the late 1940s, was incorporated into growth models based on Harrod's idea of the interaction of the multiplier and the accelerator in the 1950s and 1960s, and, finally, was codified in the textbooks on macrodynamics and growth in the 1970s.[2] Besomi also notes that the discrepancy between the story told by textbooks and Harrod's work "does not seem to have attracted the attention of recent interpreters" (Besomi 2001, p. 80). Despite Besomi's work having been well received among historians of economics, the image of Harrod as the "founder of growth theory" is still widely spread among economists.

Where Besomi focuses on Harrod's work and develops the story of his "mistaken attribution" as an epilogue to his analysis of the development of Harrod's dynamics, we focus on Solow's interpretation and its prehistory. We start with a close reading of Solow (1956),

[2] See Besomi (1998), Besomi (1999) and especially Besomi (2001). Other authors who have pointed to misrepresentations of Harrod's work are Kregel (1980) and Asimakopulos (1985).

comparing it in detail to Solow's representation of Harrod's (1939) "Essay in Dynamic Theory." To approach the question how it was possible for Solow to interpret Harrod in the way he did, we turn to a variety of interpretations of Harrod's work between 1939 and 1956. We argue that there existed a "culture of misunderstanding": in the 1940s *modeling* became the dominant way to do economics and promoted the transformation of Harrod's dynamic theory into a model of long-run growth, as it was finally presented in Solow (1956). Solow's interpretation quickly became the canonical account of "Harrod's model."

1. Solow's Harrod-Domar Case

Tagged as "the Harrod-Domar line of thought," "the Harrod-Domar model" and "Harrod's model," Harrod's analysis provides the foil against which Solow displays the power of his own simple model of long-run economic growth. Harrod's "Essay" pointed to the pervasive instability in macroeconomic dynamics, which Solow characterized with a compelling metaphor: "The . . . conclusion of the Harrod-Domar line of thought is that even for the long run the economic system is at best balanced on a knife-edge of equilibrium growth" (Solow 1956, p. 65).[3] Solow frames his own model as the repudiation of Harrod's knife-edge.

"All theory," Solow maintains,

> depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive. A 'crucial' assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic. When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious, the results are suspect. [p. 65]

[3] When no confusion arises, references to Solow (1956) and Harrod (1939) are by page number only, omitting author and publication date.

Solow claims to accept "all the Harrod-Domar assumptions" except fixed proportions (p. 66). We can see what Solow regards as Harrod's (and Domar's) assumptions by looking at what he himself assumes for his own model (pp. 66-68):

1) an economy with single-commodity (Y);
2) a constant savings rate (s), so that savings $S = sY$;
3) a constant-returns-to-scale production function with smooth substitution between capital and labor;
4) labor that grows at an exogenous rate (n);
5) "no scarce nonaugmentable resource like land";
6) flexible prices and wages;
7) constant full employment of factors of production;
8) and closely related to this last assumption, the identity of *ex ante* and *ex post* investment, which guarantees the identity of *ex ante* investment and savings, allowing the accumulation of capital to be described by the savings function alone.

The "crucial assumption," the one that distinguishes Solow's stable model from Harrod's knife-edge model, Solow believes, is assumption 3. Harrod's contrasting assumption of a fixed-proportions production function is not "reasonably realistic."

Let us see how Solow exposits "the Harrod-Domar case" as a special case within his framework. It is important to remember that we are explaining the connection between Solow's and Harrod's frameworks from the point of view of Solow's model and interpretation. Harrod, as we shall argue, would reject this interpretation – particularly the move of reducing the analysis to a question of the shape of the production function. Solow proposes that Harrod's analysis is grounded in a *fixed-proportions* production function

(1) $$Y = F(K,L) = \min\left(\frac{K}{a}, \frac{L}{b}\right),$$

where Y = output, K = capital, L = labor, and a and b are production parameters. This function exhibits constant returns to scale, but it does not allow substitution between capital and labor. The production function can be recast into what has been called subsequently "intensive form":

(2) $$y = f(r) = \min\left(\frac{r}{a}, \frac{1}{b}\right),$$

where $y = Y/L$, $r = K/L$ and $f(r) = F(K/L, 1)$.

Earlier in the article, Solow had worked out the dynamics of growth in any system with a constant-returns production function, yielding his "fundamental equation" (p. 69):

(3) $$\dot{r} = sf(r) - nr\,.$$

The time rate of change of capital per worker ($\dot{r} = \frac{dr}{dt}$) is the difference between savings per worker ($sf(r)$)and the amount of additional capital needed to outfit a growing number of workers with the current rate of capital per worker (nr). In terminology that gained currency later, *capital deepening* ($\dot{r}$) is the difference between additions to capital in the form of savings and *capital widening* (nr).[4]

Figure 1 – which is not Solow's own but is closely related to his Figure IV (p. 74) – uses equations (2) and (3) to present the dynamics of growth graphically. Panel A shows production, which rises as r increases at a rate of $1/a$ up to the point that $r = a/b$, where it becomes horizontal at an output $y = 1/b$. To the left of a/b, capital constrains output and there is some unemployment

[4] The term "capital deepening" is not mentioned in Solow's "Contribution," but he mentions it in a letter to Eisner the same year: "From a different point of view, if we imagine a society committed to maintain full employment at every moment of time, there will still be a multiplicity of ways this can be accomplished, some with much investment and some with little. Now Harrod is a subtle enough man to see this. He does make a few remarks to that effect in his book, as your quotes show. (By the way, the word 'deepening' goes well beyond this literature to Hawtrey and perhaps earlier.)" (Solow to Eisner, June 14, 1956, Solow Papers, Box 54, File E: 2 of 2). Domar, in the context of his work on economic growth and employment, also speaks of "deepening of capital" (Domar 1946, p. 142).

of labor; to the right of a/b, labor is fully employed and additional capital is redundant. The savings function $sf(r)$ is a simple scaling of the production function. It also has a kink at a/b where savings per worker $S/L = s/b$. The needs of capital widening are shown as rays from the origin with a slope n. Consider the particular growth rate of the labor force n_1. Its ray, labeled n_1r, intersects the savings function at $r = \frac{s}{n_1 b}$.

According to equation (3), the difference between the savings function and the capital-widening ray determines the time rate of change of capital, which is shown as the phase diagram in panel B. The difference reaches a maximum at $r = a/b$, falls to zero at, and then become negative to the right of $r = \frac{s}{n_1 b}$. The arrows indicate that, for any value of r less than $\frac{s}{n_1 b}$, r increases and, for any value greater, r decreases. While the phase diagram is cast in terms of the time rate of change of r, the growth of output per worker ($\frac{\dot{y}}{y}$) is itself completely determined by the time path of r.

The growth rate of the labor force may be high enough (or, equivalently, the savings rate may be low enough) that the nr ray never intersects the savings function. The ray n_2r illustrates such a case, and panel B shows the corresponding phase diagram. In such cases, r collapses toward zero, no matter what positive value it takes at the start.

Solow relates his model to Harrod's in the following way. First, for Harrod (1939, p. 30), the natural rate of growth is given by the maximum rate permitted by the rates of growth of population, technology, and labor-force participation. Thus, in a model without technical progress and a constant participation rate, the natural rate of growth $G_N = n$ unambiguously, and the slope of the nr ray is, in fact, the natural rate of growth. Solow interprets Harrod's warranted

rate of growth as the growth rate dictated by the production function and the savings propensity. From equation (1), the time rate of change of output $\dot{Y} = \frac{\dot{K}}{a}$. Since $\dot{K} = sY$, the warranted rate of growth $G_W = \dot{Y}/Y = \frac{sY/a}{Y} = \frac{s}{a}$. And since the incremental capital-output ratio (C) is the inverse slope of a ray from the origin to the point of production, $C = a$ for the production function (1). Substituting yields Harrod's own "Fundamental Equation":

(4) $$G_W = \frac{s}{C}.$$

The slope of the savings function to the left of a/b in Figure 1, panel A, therefore, gives us the warranted rate of growth.

To understand Solow's analysis of the knife-edge, consider a case in which ray n_3r coincides with the upward-sloping portion of the savings function as shown in Figure 1, panel A (the corresponding phase diagram is in panel D). Start with the economy at a full-employment, moving equilibrium in which it is growing at a warranted rate exactly equal to the natural rate ($G_W = G_N$). Any change of parameters that breaks the equality of the warranted and natural rates results in divergent movements of r. For example, an increase in the growth rate of labor to, say, n_2 (equivalent to a decrease in s) moves the economy from the phase diagram in panel D to that in panel C, which indicates the inexorable collapse of r, as the economy *relatively* disinvests.[5] On the other hand, a decrease in the growth rate of labor to, say, n_1 moves the economy to phase diagram B. The growth rate of output is unaffected, but *redundant* capital accumulates until the economy comes to rest again at $r = \frac{s}{n_1 b}$. In either of these extreme adjustments, the warranted

[5] Disinvestment is only *relative* to the size of the labor force because capital is permanent and there is no depreciation in the model.

and natural rates are permanently pulled apart. And any step away from their initial equality results in one of the extreme adjustments, which, according to Solow, define the knife-edge.

Solow attributes the knife-edge phenomenon to the fixed-proportions production function. To see why consider the analogous diagram to Figure 1 when capital and labor are smoothly substitutable. In Figure 2, panel A, neither the production function nor the savings function has a kink, and both are concave to the r axis (compare to Solow's Figure I (p. 70)). The capital-widening ray $n_1 r$ is shown cutting the savings function from below. The positive intersection at $r = r^*$ corresponds to a growth rate n_1. The warranted rate of growth is given by the slope of a ray from the origin to the intersection of $sf(r)$ and $n_1 r$. Since this ray must coincide with $n_1 r$, the warranted and natural rates of growth are equal. The phase diagram in panel B shows that any small deviation of r from r^* will reconverge on r^*; and, although the warranted and natural rates of growth may temporarily diverge, they cannot be pulled permanently apart. Any small change in n or s changes the location of r^*. Again, warranted and natural rates of growth would temporarily diverge, but would reconverge on n over time. There is no *excess* capital at r^*.

While there is no knife-edge in Figure 2, Solow does not over-claim: "There *may not be* – in fact in the case of the Cobb-Douglas function there never can be – any knife-edge" (p. 73, emphasis added). A knife-edge could still occur if the capital-widening ray rose faster at the origin than the savings function as shown with the ray $n_2 r$ in Figure 2 (compare to Solow's Figure III (p. 72)). The phase diagram in panel C shows that for any initial r, progressive collapse towards zero capital and output is the only possible outcome. Such a knife-edge cannot

occur with the Cobb-Douglas production function: since it has an infinite slope at the origin, any ray with a finite, positive slope must cut it from below.[6]

2. Harrod's Dynamics

Where does Solow go wrong?

In one sense, everywhere. Solow claims that his model differs from Harrod's *only* in not assuming a fixed-proportion production function. In fact, one could make a case that Harrod does not subscribe to *any* of the assumptions numbered 1–8 in Section 1 above. But these are not all "crucial assumptions" in Solow's sense, and we concentrate on the fundamental differences.

2.1 HARROD'S INSTABILITY

The *first* fundamental difference is that Harrod and Solow address different conceptual problems. Solow's article is entitled "A Contribution to the Theory of Economic Growth," and that title accurately conveys Solow's preoccupation with long-run economic growth. Solow self-consciously distinguishes between Keynesian pathologies and a world of perpetual full employment:

> Everything [in my model] is the neoclassical side of the coin. Most especially it is full employment economics – in the dual aspect of equilibrium condition and frictionless, competitive, causal system. All the difficulties and rigidities which go into modern Keynesian income analysis have been shunted aside. [Solow p. 91]

Solow singles out r^* as the "equilibrium," where r^* is the point at which, after all adjustments are done, the economy achieves a steady rate of growth equal to the natural rate – a long-run equilibrium – even though at every point along the adjustment path supplies equal demands and

[6] The "Inada conditions" later provided a set of sufficient regularity conditions to guarantee the existence, uniqueness, and stability of a well-behaved steady-state equilibrium in which no knife-edge phenomena can occur (Inada 1963).

saving and production plans are all satisfied, so that there is never any disequilibrium in a wider sense.

Harrod's article, in contrast, is entitled "An Essay in Dynamic Theory." The implicit contrast is not between short-run and long-run or between full employment (neoclassical) and less than full employment (Keynesian) but between dynamic and static:

> Static theory consists of a classification of terms with a view to systematic thinking, together with the extraction of such knowledge about the adjustments due to a change of circumstances as is yielded by the "laws of supply and demand.". . . [Dynamic] "theory" would not profess to determine the course of events in detail, but should provide a framework of concepts relevant to the study of change analogous to that provided by static theory for the study of rest. [Harrod p. 14]

Harrod defines "dynamic" broadly "as referring to propositions in which a rate of growth appears as an unknown variable" (p. 17)[7]. Thus, dynamics includes more than models of economic fluctuations, such as formal multiplier-accelerator models in which dated variables and explicit lags play a crucial role. He regards such models as perhaps explaining oscillations about trends, but he also suggests that oscillations in the trend itself are a crucial part of dynamics (p. 15). In contrast to Solow's equilibrium at r^*, in which growth had settled in to a steady rate, Harrod holds that

> [t]he line of output traced by the warranted rate of growth is a moving equilibrium, in the sense that it represents the one level of output at which producers will feel in the upshot that they have done the right thing, and which will induce them to continue in the same line of advance. [Harrod, p. 22]

The equilibrium is moving, not only in that the economy on a warranted path is growing, but also in that the parameters that govern the warranted rate itself may change frequently without any sense of convergence to a steady-state "equilibrium" of Solow's type. Indeed, Harrod's object is

[7] There was some ambiguity with regard to Harrod's definition of dynamics: At another place in the "Essay" Harrod defines dynamic theory as dealing with situations in which "certain forces are operating steadily to increase or decrease certain magnitudes in the system" (Harrod 1939, p. 14).

not to analyze a particular path *ceteris paribus* for the economy, not to compare the values of variables at different periods, but to elucidate the forces that may systematically drive the economy away from its equilibrium, warranted path *at any particular time* (pp. 17, 24-25). Growth is a central concern, since growth and change are essentially synonyms, but *long-run* economic growth of the type that animates Solow simply does not define the agenda.

The *second* fundamental difference between Solow's interpretation and Harrod's essay is that Harrod makes no explicit assumptions about a production function, and his implicit assumptions do not warrant a fixed-proportions, constant-returns-to-scale production function, such as equations (1) or (2). Recall the previously cited methodological preamble to Solow's essay: "The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the results are not very sensitive" (p. 65). His assumption of an economy with a single good is, perhaps, one of these inevitable simplifying assumptions, but it is not one that Harrod shared:

> [aggregate output is] compounded of all individual outputs. I neglect questions of weighting. Even in a condition of growth, which generally speaking is steady, it is not to be supposed that all the component individuals are expanding at the same rate. [Harrod, p. 16]

Harrod saw aggregate output as a summary statistic and not as a single commodity. This might appear to be a small difference. Solow did not literally believe that the economy produced a single commodity; rather he made a strong, simplifying assumption. The importance of the difference becomes clear in Solow's derivation of the knife-edge from the sharply defined parameters of the production function. The capital-output ratio (C) is a fixed parameter (a in equations (1) and (2)) for Solow. It is not fixed for Harrod:

> The value of C depends on the state of technology and the nature of the goods constituting the increment of output. It may be expected to vary as income grows and in different phases of the trade cycle; it may be somewhat dependent on the rate of interest. [p. 17]

> *C* may also be expected to vary with the size of income, e.g., owing to the occurrence of surplus capital capacity from time to time . . . [p. 25]

While the capital-output ratio is not constant for Harrod, it is importantly independent of the actual rate of growth. Its independence is related to the distinction between the actions of the individuals and their aggregate consequences, a well known Keynesian trope that frequently appears in discussions of fallacies of composition (Harrod pp. 22-25). While the aggregate balance is governed by the warranted rate of growth (and, therefore, by *C* and *s*), the actual rate of growth is governed by the reactions of many individual producers to particular conditions of over- or under-production. Individually rational responses to these particular conditions drive aggregate actual growth away from the warranted path. *Harrod's instability* is, then, *pace* Solow, not an unstable relationship between the warranted and natural rates of growth but an unstable relationship between the warranted and *actual* rates of growth.

Where Solow's own model involves an endogenous adjustment of the capital-output ratio to that required by the natural rate of growth, endogenous adjustment is not critical for Harrod. It is not, however, that he denies the possibility. *C* is not a constant; it may respond over time to excess capacity or other factors. Still, what matters to Harrod is its value at a point of time. He is not primarily interested in the specific paths of output or in the long-run consequences of some change in parameters or policy action. Rather he wants to point out the difficulty in staying on a warranted path at any moment. Where Solow emphasizes the ultimate consequences of the knife-edge, Harrod emphasizes only how hard it is to stand on such a narrow support. Solow (p. 91) introduces the metaphor of a tightrope as an alternative to the metaphor of the knife edge[8]. As Solow conceives of the problem, the question of interest is where does the tightrope ultimately

[8] Besomi mentions Yeager's (1954) "nervous tightrope walker" as the forerunner of Solow's knife-edge (Yeager 1954, p. 59, cited after Besomi 1999, p. 204).

lead; as Harrod conceives of the problem, the question of interest is how difficult it is for the tightrope walker to keep his balance.

The second fundamental difference between Solow's interpretation and Harrod's essay is closely related to a *third* – and probably most fundamental – difference: for Solow *ex ante* savings and investment are always equal to *ex post* savings and investment. "A remarkable characteristic of the Harrod-Domar model," Solow writes "is that it consistently studies long-run problems with the usual short-run tools" (p. 66). This is not a characteristic of Harrod's analysis – remarkable or otherwise – because, as we already showed, Harrod is not concerned with long-run problems, but with the instability of the actual growth rate relative to the moving equilibrium (warranted) growth rate – a short-run problem for which he believes short-run tools are appropriate. Solow contrasts the warranted and the natural growth rates; Harrod mainly contrasts the warranted and the actual rates. It is seventeen pages into a thirty page essay before Harrod so much as mentions the natural rate of growth, and, by that point, the core analysis is complete.

Harrod defines the incremental capital-output ratio (C) as

> that addition to capital goods in any period, which producers regard as ideally suited to the output which they are undertaking in that period. . . [T]he term *ex ante* . . . will be used in this sense. [p. 19]

Harrod proceeds in the standard Keynesian manner: when *ex post* investment falls short of *ex ante* investment, output is stimulated as producers react to unexpected and undesired reductions of inventories ("stocks" for Harrod), and conversely when *ex post* investment exceeds *ex ante* investment. Harrod's principal mechanism is "a marriage of the 'acceleration principle' and the 'multiplier' theory" (p. 14). It is not, as Solow suggests, the wrong tool for the job; it is a different tool for a different job.

While Solow is aware that he has banished Keynesian problems from his own model by assuming constant full employment and the equivalence (not just the *ex post* equality) of savings and investment, he does not acknowledge that it is this assumption rather than the substitutability assumption that ultimately separates his model from Harrod's theory (Solow, p. 91).

The key role of the assumption that *ex ante* and *ex post* investment are constantly equal can be clarified by reflecting on so-called optimal growth models, now a standard part of the first-year graduate curriculum in economics (see Blanchard and Fischer 1989, ch. 2, for one of many textbook expositions). Optimal growth models are essentially Solow's neoclassical growth model in which the savings rate is no longer given parametrically, but is the endogenous outcome of an intertemporal utility-maximization problem. The phase diagrams for such models typically involve not just capital per worker (r) but also consumption per worker. And typically, they are everywhere unstable except for a unique saddle path. The perfect foresight, or rational-expectations, solution simply assumes that a rational agent would ignore all other paths and jump to the saddle path. The saddle path is precisely the set of points for which *ex ante*, planned investment and consumption decisions align with *ex post*, realized investment and consumption. Growth along the saddle path is growth at the warranted rate, even though that rate (as it does on the phase diagram in Figure 2, panel B) changes constantly until it reaches the steady state. But what happens if we do not assume perfect foresight or rational expectations? Then, the economy is, except by sheer luck, in the unstable part of the phase space. The actual growth rate differs from the warranted rate – an instability of Harrod's, not Solow's, form, despite the fact that the production function is one with smooth substitutability.

What if we maintain perfect foresight or rational expectations but replace the production function with a fixed-proportions function such as equation (2)? The model still determines a

phase space unstable everywhere except a saddle path, and the solution is to jump directly to the saddle path. The key assumption is not the substitutability of factors. The key assumption is the equivalence of *ex ante* and *ex post* investment.

The *fourth* fundamental difference between Solow's interpretation and Harrod's essay is that Harrod is that Solow's knife-edge and Harrod's instability along the warranted growth path are almost completely unrelated ideas. As Solow describes it:

> Were the magnitudes of the key parameters – the savings ratio, the capital-output ratio, the rate of increase of the labor force – to slip ever so slightly from dead center, the consequence would be either growing unemployment or prolonged inflation. [p. 65]

As we have already seen, a knife-edge phenomenon is not unique to the fixed-proportions production function. Compare the phase diagrams (panels B and C) in Figures 1 and 2. They are topologically identical. In each case, if the *nr* ray rises faster at the origin than the production function (panel C), then there is a progressive collapse until growth and production disappear. In each case, if the *nr* ray cuts the savings function from below, there is progressive expansion until technology limits output to factor capacity and growth to the natural rate. There is one salient difference: in Figure 2 with substitutable factors, full employment is maintained along the adjustment path; while in Figure 1, unemployment rises in panel C and capital is unemployed at the steady state in panel B. Of course, in Figure 2, panel C, unemployment is effectively 100 percent in the long-run state (at the origin), even if it is zero on the transition path. And if Harrod is to be believed and *C* adjusts to surplus capacity (p. 25), the excess capital in Figure 1, panel B, will not persist.[9]

[9] It is hard to see why the steady state in Figure 1, panel B, should be characterized as contributing to "prolonged inflation," as Solow suggests, especially in a model in which there is no money and no general price level. There is not even excess demand at the steady-state. There is excess accumulation of capital, and that, rather than, inflationary pressure seems to be the proper analogy to the increasing unemployment in panel C.

The considerations so far demonstrate that some of Solow's knife-edge properties do not depend principally on the assumption of a fixed-proportions production function. In particular, whether the economy away from the natural rate of growth grows to an upper or lower bound appears to be independent of substitutability. Rather it depends on regularity conditions in the production function – at the lower end, principally whether the slope of the production function is vertical at the origin. The existence of surplus factors of production is a difference that can be more clearly attributed to nonsubstitutability; but, as we have already seen, Harrod does not appear to subscribe to a fixed-proportions production function.

More fundamentally, Solow's knife-edge is not the same as Harrod's instability of the warranted rate of growth. Solow's analysis of the knife-edge concerns the failure of the warranted rate to adjust to the natural rate, where Harrod's instability concerns the divergence of the actual rate of growth from the warranted rate. The radical difference between them is clear from the fact that they work in opposite directions.

As we showed in Section 1, if an economy with a fixed-proportions production function starts with equality between the warranted and natural rate and if that equality is upset, for example, by a fall in the savings rate (s), which lowers the warranted rate (s/C), then the economy as described in the phase diagram in Figure 1, panel C, crashes towards zero output. This is one side of Solow's knife-edge.

Consider, Harrod's analysis of the same fall in the savings rate. The fall in the savings rate reduces the warranted rate of growth below the actual rate of growth. "Savers will find that they have saved more than they would have done had they foreseen their level of income . . . Consequently they will be stimulated to expand purchases, and orders for goods will consequently be increased" (Harrod, p. 21), which widens the divergence between the actual rate

of growth G and the warranted rate G_W. Whereas in the case of Solow's knife-edge, a fall in the savings rate directs the economy toward lower levels of output, in the case of Harrod's instability, it directs the economy toward higher levels of output. Solow's knife-edge and Harrod's instability concern the relationship of different growth rates (G_W and G_N for Solow; G and G_W for Harrod); and, for any change in parameters (s, n, C), they work in opposite directions. They address nearly orthogonal issues.

Harrod's instability can arise in Solow's own model provided that *ex ante* and *ex post* investment is allowed to diverge. Consider the phase diagram in Figure 3, which corresponds to panel B in Figure 2. In the usual analysis of Solow-type neoclassical growth models, the economy is restricted to paths along the capital-adjustment curve. Consider what happens when the economy is at the steady state r^* and the savings rate falls. The curve shifts down, and normally the analysis proceeds by finding a point on the new curve directly below r^*. This point is now out of steady state, but the arrows along the curve indicate that capital would be relatively depleted until a new steady state where $\dot{r} = 0$ was reestablished. But what justifies the jump from the initial point to the new curve? Only the insistence that there be no divergence between *ex ante* and *ex post* – rationalized by an assumption such as perfect foresight or rational expectations. If, as Harrod presumes, we can make no such assumption, then the economy is left at a point in the phase space (r^*, $\dot{r} = 0$) that is above the new capital-adjustment curve, and all points above this curve are driven to the right (and all below to the left) by the interaction of the "marriage of the 'acceleration principle' and the 'multiplier.'" The capital-adjustment curve is thus seen to be a saddle-path, similar to what one typically finds with neoclassical optimal growth models.

If we interpret, the capital-adjustment curve in Figure 3 as we did in Section 1 as governing the transient movements in the warranted rate of growth as it adjusts toward the natural rate, then Harrod can be said to have directly described the situation analyzed using Figure 3 in terms that accurately reflect the way it is drawn. A fall in the savings rate is a Keynesian stimulus. Harrod writes:

> Suppose that one of these stimulants begins to operate when the actual rate is equal to the warranted rate. By depressing the warranted rate, it drags that down below the actual rate, and so automatically brings the actual rate into the field of centrifugal forces, driving it away from the warranted rate-that is, in this case, upwards. Thus the stimulant causes the system to expand. [p. 31]

2.2 The Constitutional Weakness of the Macroeconomy

So far, we have argued that Solow's criticism of Harrod misfired because the key difference between them was found not in assumptions about substitutability in the production function but in the assumption that *ex post* and *ex ante* quantities are always equal. Harrod's instability is, therefore, the instability of the actual growth rate relative to the warranted rate, whereas Solow's knife-edge is the instability of the warranted rate relative to the natural rate. Yet, Harrod does consider the relationship of the warranted and natural rates; does that not lend some support to Solow's analysis? We think not.

As mentioned before, Harrod considers the natural rate only late in the paper, after his main analytical conclusions have been established. Solow locates the knife-edge phenomenon in any case in which the warranted and natural rates fail to converge, so that the possibility that there is a persistent gap between warranted and natural rates, which Harrod seems to take as given, is exactly what Solow rejects. Yet, as we have already seen, there need be no persistent gap for Harrod's instability to arise. The message of Figure 3 is that, even if over time warranted

and natural rates converge as in Solow's model, any change in parameters that temporarily drives them apart sets up Harrod's instability – with or without a fixed-proportions production function.

Solow is, of course, correct that Harrod does not see the convergence of the warranted and natural rates as given: "There is no inherent tendency for [the warranted and natural] rates to coincide" (Harrod, p. 30). Their coincidence, Solow says, would be just "an odd piece of luck" on Harrod's view (p. 77). In one sense, that is right, but probably not for Solow's reasons. Since Solow sees the natural rate as evolving slowly and the warranted rate in his account of "Harrod's model" as governed by fixed parameters, only if those parameters happened to be just right could the two rates coincide. Harrod himself rejects the fixed parameter view: "Indeed, there is no unique warranted rate; the value of the warranted rate depends upon the phase of the trade cycle and the level of activity" (p. 30). Again, it is clear that Harrod's concern is not with long-run economic growth.

Harrod's concept of full employment is unlike recent macroeconomic analyses that relate full employment or potential output to something like Milton Friedman's natural rate of unemployment. The popular concept allows output to run above, as well as below, full employment or potential. In contrast, Harrod's conception of full employment is the typically Keynesian notion of a ceiling. An economy is fortunate if it operates close to the ceiling, but it can never operate above it. The economy cannot grow faster than the natural rate allows in the long run – that is, it cannot operate above full employment, even though it may grow faster than potential output when it starts below full employment. Harrod introduces a fourth rate of growth to his famous three: actual, warranted, and natural. The *proper warranted rate of growth* is "that warranted rate which would obtain in conditions of full employment" (p. 30) – that is, the rate of growth of potential output.

Harrod conjectures that the relationship between the proper warranted rate and the natural rate determines the *likelihood* of the economy operating below full employment for any length of time. He writes:

> The system cannot advance more quickly than the natural rate allows. If the proper warranted rate is above this, there will be a chronic tendency to depression; the depressions drag down the warranted rate below its proper level, and so keep its average value over a term of years down to the natural rate. But this reduction of the warranted rate is only achieved by having chronic unemployment.
>
> The warranted rate is dragged down by depression; it may be twisted upwards by an inflation of prices and profit. If the proper rate is below the natural rate, the average value of the warranted rate may be sustained above its proper level over a term of years by a succession of profit booms. [p. 30]

Harrod's conjecture amounts to a claim that when the warranted rate of growth exceeds the natural rate of growth the economy displays a *constitutional weakness*: a tendency towards frequent recessions.

Harrod's thinking can be understood with the help of Figure 4, which shows the time path of output (Y) on a logarithmic scale, so that growth rates correspond to the slopes of the curves. In panel A, up to time t_0, the proper warranted rate and the natural rate coincide. At t_0, the proper warranted rate increases (s increases or C falls) as shown by the upper, gray path. Since the natural growth path is the full employment path, actual output cannot move into the area between the natural and warranted growth paths. Harrod's instability can manifest itself only in the downward direction as shown by the arrows.

In contrast, panel B shows a case identical up to t_0, in which the proper warranted rate falls at t_0 (s falls or C rise). Harrod's instability can still manifest itself downward, but the region above the proper warranted growth path and below the natural growth path is now feasible, so that it may manifest itself upwards as well. Instability can, in this case, drive the economy toward full employment and keep it there. Recessions are possible in both cases; but, when the

proper warranted rate exceeds the natural rate, they are inevitable; and, when the natural rate exceeds the proper warranted rate, full employment may prove a common outcome. Harrod's conjecture is that when the proper warranted rate exceeds the natural rate, the economy will be constitutionally weak in the sense that recessions are more frequent; whereas when the natural rate exceeds the proper warranted rate, the economy will be stronger, with fewer recessions and more time spent near full employment.[10].

3. Cultures of Understanding

We have argued that Solow's Harrod is a misreading in that Harrod did not aim at a model of long run growth and did not assume a constant capital coefficient. How was such a misreading possible? Looking only at Solow's and Harrod's articles side by side, one might well wonder whether Solow's interpretation of Harrod reflects a reading of Harrod at all.

We would like to suggest that there are, in effect, different interpretive communities, each with its own interpretive practices and methodological and substantive presuppositions. Within each interpretive community there are common practices of doing research and a certain "culture of understanding" that implicitly guides its work. Thus, what was "in the air" with respect to Harrod was his work as interpreted through the culture of understanding of Solow's interpretative community. The canons of an interpretive community may be explicit, but more often are tacit and hardly noticed by its members.[11]

[10] In a related unpublished paper, Hoover (2008) provides a preliminary test that favors Harrod conjecture for U.S. data.

[11] The notion of cultures of understanding bears a family resemblance to Fleck's (1935) concept of thought collectives, Kuhn's (1962) paradigms, and Lakatos's (1978) methodological research programs. While we accept the genuine resemblance of these ideas to our own use of a "culture of understanding," we reject any extreme reading of them that suggests insuperable incommensurability – rather than intrinsic difficulty in communication – across communities. "Misreadings" and "misunderstandings" as described above are an intrinsic and prevalent feature of science as a cultural and social enterprise. Depending on the closeness and overlapping of communities and their

Baumol implicitly underlines the role of an interpretive community guiding the reading practices of economists: "later writers are responsible only to be careful to assert that they have merely taken off from what Harrod's writing suggested to them, not that they are correctly reporting his ideas" (Baumol 2000, p. 1037). The "main achievement of [Harrod's] model lies in the ideas it inspired in those who did not fully understand it" (Baumol 2000, p. 1039). Baumol's view makes the actual development of a discipline excessively linear and, even while ostensibly promoting an instrumental or utilitarian view of the history of economics, in fact saps that history of many of its instrumental resources as well as of its rationale, which is to tell true stories (see Hoover 2004, section I).

In reality, interpretive communities are not isolated and impermeable; rather they overlap and interpenetrate – both intellectually and temporally. There is, therefore, commerce among interpretive communities and the possibility of translation between the interpretive frameworks of distinct communities. It is too glib to suggest – as Baumol does – that the value of one community lies entirely in what inspiration it may have provided for a dominant community. We must recognize that from Harrod's point of view, Solow's interpretation is the product of a culture of misunderstanding. Translation is possible; accuracy in translation should be valued; yet, it is likely that something of value can be lost in translation. We locate Solow's reading of Harrod in the difficulties of commerce and translation between two identifiable, yet nonetheless, not mutually isolated, interpretive communities.

3.1 THE "HARROD-DOMAR LINE OF THOUGHT"

practices difficulties in communication might be overcome more easily or pose serious difficulties of grasping the meaning of products of a different community. See Fish (1980) for the notion of interpretive communities and Hoover (1991, 1994) for arguments against insuperable incommensurability.

Besomi demonstrates that the interpretation of Harrod's work as a theory of growth developed in the 1940s, when the discussion was fueled by the publication of Harrod's *Towards a Dynamic Economics* (1948) and of Domar's (1946) article in *Econometrica.* Domar stressed the similarity of Harrod's fundamental equation to his own (Besomi 2001, p. 79). Harrod belonged to the English Keynesian school. Its central concern in the decade before World War II had been with the problem of the trade cycle, motivated, of course, by the experience of the Great Depression. Harrod, like Keynes, pursued a Marshallian macroeconomics. Methodologically, it favored using mathematics in limited ways to isolate the qualitative causal structure of the economy. Keynes understood dynamics to be a matter of isolating and cataloguing the casual forces that changed important economic variables (Hoover 2006). His *General Theory*, however, downplayed the forces of change in favor of articulating a new architecture for thinking about the economy as a whole. Harrod's "Essay" is, in part, a reintroduction of the explicit dynamic concerns of Keynes's earlier *Treatise on Money* (1930, p. 120) into the new macroeconomic framework. Growth was simply not the central issue.[12]

Solow's approach is also distinguished from Harrod's by its commitment to the practice of formal modeling. It is easy to forget that *models* as primary instruments of science are largely a post-World War II phenomenon. A crude measure of the rise of modeling is given by the fact that the frequency of the use of "model" or "models" rises in economics journals by 300 percent between the ten years before and the ten years after World War II. A smaller, but similar, increase also shows up in science journals.[13] The transition of economics into a modeling science

[12] For a treatment of the Harrod-Keynes correspondence see Kregel (1980) and especially Besomi (1995); on Harrod's 1938 draft of the "Essay" see Besomi (1996) and on the changes he added reacting to Keynes' and Marschak's criticism see Sember (2010).

[13] Data are drawn from searches on JSTOR archive, executed on 25 November 2012, for the use of "model" or "models" for 173 economics journals and 29 general science journals, counting the number of articles using one of

had, of course, already begun before the war, and was epitomized in the work of Frisch, Tinbergen, and Samuelson.[14] Harrod was aware of this work, as we noted in section 2, and explicitly sought to distinguish the questions that he wanted to address in his "Essay" from the new emphasis on models consisting of formal systems in which the dynamics were driven by an explicit lag structure. Although the American growth economics of the 1950s represents a distinct community from the English trade-cycle economics of the 1930s, it took some time for the distinctions to emerge. Over the period between the late 1930s and the early 1950s, economists – the older Marshallian macroeconomists as well as the new formal modelers – investigated the implications of combining the consumption multiplier and the investment accelerator. And it is in this literature that we find the first traces of Solow's representation of Harrod.

When reading all the papers about growth and dynamics between 1939 and 1956 in which Harrod is mentioned, it is particularly striking that almost all of them express some kind of doubt as to "what Harrod actually meant" with regard to various topics. Thomas Schelling (1947, p. 867) for example notes that "whether [Harrod] meant to imply that it [non-growth-induced investment] actually would be proportionate to income, or only meant to give it formal expression, is not clear." Elsewhere Schelling (1947, p. 868) mentions that Harrod's warranted rate of growth appears "to us as two definitions whose equivalence is seriously in question … [O]n the one hand, they feel that they have done the right thing; on the other hand, they are induced to 'continue in the same line of advance.'" In a similar vein, Baumol (1948, p. 507, fn. 1) notes that "Mr. Harrod himself sometimes employs the one assumption [of saving as a

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the search terms as a percentage of all articles. For economics journals: 1930-39 = 14 percent; 1946-55 = 42 percent. For general science journals: 1930-39 = 14 percent; 1946-55 = 23 percent.

[14] Frisch (1933), Tinbergen (1939) and Samuelson (1939)**;** also see discussions in Louçã (2007) and Morgan (2012, ch. 6).

constant proportion of this period's income] and sometimes the other [of savings as a constant proportion of the preceding period's income]." McCord Wright (1949, p. 326) shows Harrod using six independent qualifications regarding the warranted rate of growth. These doubts are also to be found in Solow's correspondence: Three years before the publication of his "Contribution" Solow "confesse[s] to a certain uneasiness in this Harrod maybe-maybe land of equilibrium growth, in which one never knows anything about the behavior-dynamics of the system. But as long as we work in this genre, we're stuck" (Solow to Johnson, September 28, 1953, Solow Papers, Box 56, File J: 1 of 2). Here, Solow demands a more clear-cut analysis, something he also recalls in an unpublished introduction to Harrod's 1946 lectures: "[Harrod's] 'theorems' will seem odd to a modern economist. They left Harrod's contemporaries confused too; there was considerable discussion at the time about what they really meant" (Solow 2009, p. 3).

Several authors introduced different notations and formalisms to "clarify" Harrod's work and/or to represent it in a "more general" way. A common practice was to combine Harrod's "Essay" with Domar's "Capital Expansion, Rate of Growth, and Employment" (1946) and to use Domar's "clearer" notations and formalisms for representing Harrod's dynamics. The first of these combinations is to be found in an article by Thomas Schelling, already one year after the publication of Domar's (1946) article. Schelling was at this time a teaching fellow at Harvard, where Solow was then a student. Describing Harrod's theory as a system of difference equations expressed in Domar's notation, Schelling implied that Harrod's savings rate is consistent with Domar's α, thereby neglecting that α is the "marginal propensity to save" (Domar 1946, p. 140). In contrast, Harrod took into account the "fraction of income saved", i.e. the average propensity to save (Harrod 1939, p.29, cf. Harrod 1959, p. 454). By further equating Domar's σ, the

"potential social average investment productivity" (Domar 1946, p. 140), with the reciprocal of Harrod's capital coefficient[15], Schelling investigated under which circumstances a "required rate of growth," which he referred to as the "Harrod-Domar $\alpha\sigma$," tends to maintain itself (Schelling 1947, p. 872). Even though Schelling acknowledged many aspects of Harrod's work that, later, Solow neglected, he still derived the value for the "Harrod-Domar $\alpha\sigma$" to maintain full employment. Similar to Domar's, but not to Harrod's approach, Schelling looked at the requirements for a full employment equilibrium along a growth path – he did not address the instability of Harrod's moving equilibrium.[16] And so, despite explicitly recognizing some of the differences between Harrod and Domar ("It should be added here that Harrod considers the equilibrium represented by this warranted rate of growth as essentially unstable", Schelling 1947, p. 868), Schelling still defined Harrod's work as equilibrium analysis, since "Mr. Harrod begins directly by inquiring what rate of income growth would tend to maintain itself" (Schelling 1947, p. 866).

Schelling's reinterpretation of Harrod in the light of Domar laid the groundwork for the later treatment of Harrod and Domar as offering a common "model" and for interpreting Harrod's "Essay" as a "theory of growth."[17] Furthermore, Schelling supposed that Harrod proceeded from the assumption that investors as a whole projected the current rate of growth into

[15] Harrod himself pointed to the difference between his capital coefficient and Domar's σ a few years later: "[H]e [Domar] designates the potential increase of output per unit of new investment by the symbol σ. I, on the other hand, make no explicit reference to this increased productivity, but … considered how many units of new investment are required, on the assumption that the new investment is properly utilized, to produce an extra unit of output; this I designated C_r" (Harrod 1959, p. 452; cf. Hagemann 2009, p. 70).

[16] A similar argument was made by Hamberg (1952) who investigated the consequences of the full employment growth rate exceeding the rate of growth required for the full utilization of capital. He reproached Harrod for only discussing "the Keynesian case" (U > E) and not considering the full implications of the situation when the full employment growth rate exceed the full capacity one and only thinking about the inflationary aspects of this situation" (Hamberg 1952, p. 446 fn.). Hamberg, as Solow later, recognized only the possible divergence between the warranted and natural rates, thus ignoring Harrod's instability principle.

[17] See Kregel (1980) for an earlier treatment of the differences between Harrod's work and the "Harrod-Domar" model.

the future "with such confidence as to guide their decisions by that criterion alone." Hence, Harrod "did go too far in attaching motivational significance to the [warranted] rate?" (Schelling 1947, p. 870).[18] Schelling tied this interpretation of Harrod firmly to the formal dynamics of the 1930s that Harrod had explicitly eschewed. Schelling (1947, p. 872) observed that Samuelson (1939) "accepts as a postulate the very investment behavior which Harrod tried to establish" (Schelling 1947, p. 872).

A year later, Baumol published an article that specifically addressed the connections between Harrod's "Essay" and Samuelson's (1939) article on multiplier-accelerator interactions. Baumol (1948) described Harrod's work as a "dynamic model" (p. 506), regarding it as a "normative study . . . indicating the sort of conditions which must be satisfied by the course of the level of the national income through time in order for investment demand to be satisfied" (p. 516). According to Baumol, Harrod's instability principle would not follow necessarily from his premises, as it required implicit assumptions about producer's expectations and their subsequent effects on production plans. Hence, "there appear to be possible alternative situations which will be characterized by considerably less instability than Mr. Harrod's argument might lead us to expect" (Baumol 1948, p. 514). Following a similar tactic to Schelling, Baumol reinterpreted Harrod's theory in a new notation "in the interest of ready comparability with the system of Prof. Samuelson" (Baumol 1948, p. 507).

The next year, Baumol (1949) formulated a "complete model," in which he made explicit the assumptions about entrepreneurial expectations that he had previously argued were implicit

[18] This point is also made by Asimakopulos (1985): "For Harrod's purposes this moving equilibrium should have the characteristics of a trendline – it should be one that the economy would follow in the absence of disturbances. The development of the model within a Keynesian framework where uncertainty over future conditions prevailed made it difficult to explain why the entrepreneurs' investment decisions would lead the economy along such a path. Harrod, when challenged by Alexander, saw the need for an explanation, but his resort to a 'representative entrepreneur' [in Harrod (1951)] was an evasion of a real problem for his theory, not a solution" (Asimakopulos 1985, p. 633).

in Harrod's theory. Specifically, he assumed that entrepreneurs plan to increase their output at the warranted rate unless there is excess demand, in which case they plan to depart from the warranted rate by an amount sufficiently great to make up for excess demand. Baumol was convinced that it is an assumption like this that is required to make "Mr. Harrod's system work the way he says it does" (Baumol 1949, p. 629).

Once Harrod's theory had been subsumed into a formal model, it was a short step to reinterpreting it as embodying a fixed-proportions production function. Harold Pilvin (1953) offered perhaps the first such explicit interpretation in an article that is one of the few mentioned in Solow's 'Contribution."[19] Pilvin (1953, p. 546) represented Harrod's and Domar's work by introducing a production function in order to provide a general treatment of "the growth problem" and, as with Baumol, to make explicit certain implicit assumptions in Harrod's formulations. Pilvin (1953, p. 548) referred to a "constant capital coefficient as employed by Domar and Harrod." He does not mention instability or cycles, and his major aim was to construct "a growth model of considerable generality" (Pilvin 1953, p. 546). Pilvin appears to be the first person in this literature to employ a graphical apparatus, similar to the one found in Solow's "Contribution," to represent Domar's model.

Pilvin thanked Domar for his comments on his work (Pilvin 1953, p. 545), and Domar, in turn, considered Pilvin's explicit production function as very useful by showing that "the existing growth models" are an extreme case "of the more 'normal' one – the former does not allow for substitution between factors, the latter does" (Domar 1953, p. 562). By saying that "as an analytical device, a constant input coefficient is God-sent, but it is quite a simplification and it should be used with care, particularly over longer periods of time when it is known to be subject

[19] Solow thanked John Chipman for a remark on Harrod's answer to Pilvin's article (Solow 1956, p. 83).

to change" he implied that Harrod actually assumed such a constant coefficient (Domar 1953, p.561; cf. Hagemann 2009, p. 81).[20]

The American modelers of the immediate postwar years found it essential to translate Harrod's theory into a new and formal language, assimilating it to their own culture of understanding. But to the degree that Harrod's central concerns – e.g., to identify the forces that would render explicit dynamic equations irrelevant or to isolate the constitutional weakness of the macroeconomy that made repeated slumps more likely – were omitted, a vital element of his work was lost in translation and the interpretive practices of the growth modelers unwittingly promoted a culture of misunderstanding that laid the groundwork for Solow's treatment of Harrod's "growth" model.

3.2 Solow and Growth Before the "Contribution"

In the early 1950s, Solow worked on the aggregation of Leontief's input-output tables and dynamic linear programming.[21] Solow (1952) applied Samuelson's stability framework to linear systems, studying the relationship between the existence of an equilibrium and the stability of a dynamic system (cf. Boumans 2009). Samuelson's concept of stability had been defined with respect to static equilibria. The next year, Samuelson and Solow applied concepts derived from P.H. Leslie's population mathematics to work out the concept of "balanced growth." They also "replace[d] the hypothesis of fixed proportions by the more general one that productions functions are homogeneous of first degree" (Solow and Samuelson 1953, p. 412).

[20] In his answer to Pilvin's article, Harrod criticized the use of a production function and especially the accompanying assumption that the rate of interest "will move in such a way as to change the productive process employed, causing it to move in an appropriate fashion along the curve of the production function" (Harrod 1953, p. 555). Harrod brings forward several arguments that, although the rate of interest "plays some part" in changing the capital-labor ratio, it is not as important as Pilvin thinks (cf. Hagemann 2009, p. 83 and Besomi 1999, p. 204).
[21] For the contexts and practices Solow's 1956 model emerged from, see Halsmayer (2013) and Backhouse (2012).

Having worked out dynamic versions of Leontief's input-output system, Solow (1953-1954) proceeded to identify "the Leontief and Harrod dynamic models" (cf. Boumans 2009, pp. 138ff). Having formulated Leontief's system in matrix form, he suggested to "banish the thought that *a* and *b* are matrices and x_t is a vector and think of these quantities as ordinary numbers. Then equation (I) looks suspiciously like Mr. Harrod's system. In fact, deep down, it *is* Mr. Harrod's system" (Solow 1953-1954, p. 74). Solow offered three reasons to identify the "Leontief and Harrod dynamic models": both traced out moving equilibrium output paths; both defined equilibrium by the *ex post* justification of investment plans (perpetual appropriateness of the existing capital stock to the current level of output); neither had anything to say about the actual time-path or contemplates in any detail the possibility or consequences of disequilibrium.

Although Solow conceded that Harrod was "of course, quite interested in the consequences of departure from equilibrium growth, and in the instability of the latter" (Solow 1953-1954, p. 75, fn.), he saw it as Harrod's fault not to have provided explicit causal dynamics. As a result, he regarded Harrod's "model" as a special one-commodity case of the Leontief dynamic system.

Solow's first representation of Harrod's dynamic theory equated it with a dynamical Leontief input-output system. Aggregated to a one-commodity economy with a fixed-proportions production function Harrod's dynamic theory was transformed into a model of long-run growth. Solow interpreted Harrod in the way that he must for him to make sense of it within the culture of understanding of the emergent work on formal models of growth. Yet, it is an interpretation that could only be regarded as a deep misunderstanding and, even inexplicable misrepresentation, from the point of view of Harrod's community. Solow presented Harrod having developed a moving-equilibrium output path; Harrod did not talk about long-run

equilibrium. Solow took Harrod to address trend growth; but Harrod principally addressed the trade cycle.

The advantage of Solow's interpretation is stated clearly at the end of the paper: his formal system provides the basis for a "more complete causal dynamics of the kind usual in business cycle theory. In these terms questions beginning 'What would happen if' would have exact answers" (Solow 1953-1954, p. 79). The disadvantage is that Solow's model sets aside the questions and their suggested answers that were the focus of Harrod's analysis. The loss is clear in a letter to Frank Hahn in which Solow opines that "[t]he Harrod-Domar legacy of paying attention only to equilibrium paths is by now an obstacle. All these ad hoc stability statements about what happens off such path are useless without an explicit causal dynamics." According to Solow, such explicit causal dynamics have to include "a theory of uncertainty -- and therefore, . . . a theory of investment" (Solow to Hahn, March 23, 1959,Solow Papers, Box 55, File H: 1 of 3).

Harrod would surely reject the idea that his principal concern is with equilibrium paths and not causal dynamics. In fact, he acknowledged the role of uncertainty in a theory of investment as implicit throughout his analysis. It is only through the lens of a formal model, in which these features are not easy to treat realistically, that Harrod can be regarded as having ignored them. Where Harrod had seen himself as having taken a first step toward addressing these issues, Solow sees them as issues that have become pressing only at a later stage in the evolution of growth models. For Solow, they are the next new thing; for Harrod, they are the reinvention of the wheel.

It is important to recall that interpretive communities are not hermetically sealed, but overlap, interpenetrate, communicate, and trade. Solow's interpretation of Harrod was not

uncontested at the time[22] and criticism led Solow to defend his interpretation vigorously: "I'm damned if the impression you leave [Harrod's "Dynamic Economic Theory"] with is one of flexibility. Harrod achieves this by first saying that perchance the rate of interest can't be driven down far enough …then giving the impression that what is needed is an eternally falling profit-interest rate, and then assuming that the rate of interest is constant and with it the capital coefficient. For my money this is the moral equivalent of fixed proportions. No doubt Harrod can't be held responsible for what his followers have made of the doctrine, but he certainly gave them a head start" (Solow to Eisner, May 14,1956, Solow Papers, Box 54, File E: 2 of 2). Answering Kaldor, Solow admited that "I was probably undesirably elliptical in referring to Harrod in terms of fixed coefficients. In effect [emphasis in original] that's where he comes out, but he does it by saying that V depends on the rate of interest, but the rate of interest is stuck" (Solow to Kaldor, November 9, 1959, Solow Papers, Box 57, File K: 1 of 4). Also in the correspondence between Solow and Harrod Solow insisted that Harrod used fixed proportions: "[I]t still seems to me that even when the equation is deduced on other grounds it is dependent on the tacit assumption that the marginal productivity of capital (or the capital coefficient) is a technical constant. . . No matter how I take it, I seem to find your equation to be dependent on the constancy of the social yield or capital" (Solow to Harrod, September 23,1960, Box 1, File 1960)[23].

Throughout these debates about growth and dynamics Harrod's theory was interpreted as a *model* of growth -- an attribution that Harrod himself vigorously rejected:

[22] For example, Solow's misplaced emphasis on the nature of the production function has not escaped notice. According to Burmeister and Dobell (1970, p. 41), "studying the Harrod position as if it were based essentially on a technological hypothesis about the production function, namely that it shows fixed proportions, misses the essential feature of Harrod's analysis."

[23] The authors are thankful to Roger Backhouse who brought this letter to their attention.

> many years after I had made certain formulations in the field of growth theory and after Professor Domar had made similar formulations, there began to be references to the 'Harrod-Domar model.' I found myself in the position of Le Bourgeois Gentilhomme who had been speaking prose all his life without knowing it. I had been fabricating 'models' without knowing it. (Harrod 1968, p.173)

As Besomi (2001, p. 88) shows, Harrod had consistently insisted that he was providing the "outline of a theory" and not a complete model, which he saw as requiring "special postulates and assumptions in regard to lags and coefficients, which can only be accepted subject to statistical verification" and providing a full explanation of growth. Still, the interpretation of Harrod's work as a model of growth fit the emerging practices of the economics profession from the 1940s on. Economics became a modeling science, and modeling became the "natural way" to do economics (cf. Morgan 2012, ch. 1). Seen in the light of the emerging modeling practice, Harrod's dynamic theory was understood as a model of growth.[24]

As E. Roy Weintraub (2005, p. 149) has noted, Harrod's ideas were initially thought to be quite wrong, then were thought to be quite difficult to interpret, and finally were quite ignored as a new generation of theorists rewrote economic dynamics from a mathematical perspective. From this perspective, Solow offered a "formalization" of Harrod's theory. Formalization is commonly regarded by the community of economic modellers as the replacement of verbal and mathematical theorizing by models and typically regards such translation as possible while preserving the equivalence of concepts between the original and the translation (cf. Boumans 2005). Such a view of formalization fundamentally underestimates the role of models. Reformulation of Harrod's "dynamic theory" in terms of a model – that is, translation between the practices of different interpretive communities – involved serious losses. Because they were

[24] The nearly continuous economic growth in developed economies in the early postwar years probably diminished the urgency of problems connected to the business cycle, and emphasized the need to understand how trend growth could be sustained (cf. Besomi 2001, p. 85) .

not easily modeled through a closed system of differential or difference equations, Harrod's concern with the stability of a dynamic path were not understood by the growth modelers. In identifying Harrod's instability with Solow's knife edge, Harrod's major longer-term concern with the constitutional tendency of the economy to slip into recession when savings rates were relatively high was omitted from the research agenda: the formal growth models of the 1950s and 1960s had no natural mechanism for articulating the issue that takes up much of the latter half of Harrod's "Essay." Consequently, not only Harrod's dynamic theory, but also his ultimate purpose, providing an analysis of those factors that determine the trade cycle, got lost. His work came to be defined by Solow's "Contribution" – namely, a model of the long-run with a unique stable growth path and a knife-edge (cf. Besomi 1999, p. 205, Kregel 1980, p.118)[25].

4. Harrod's Fate

Solow originally cast his model as the solution to the problem of instability in Harrod's dynamic analysis. Economic modeling became the dominant research practice of the mainstream of the economics profession. The mainstream generally accepted Solow's model, as well as his reading of Harrod, with the result that Harrod's analysis has been consigned to the dustbin of history. Textbooks present the history of their science as a linear story of progress, and rarely reflect on what might have been lost. Barro and Sala-i-Martin (2003, p. 17), for example, write of Harrod's supposed fixed-proportions growth model: "Although these constructions triggered a good deal

[25] Years later, Solow recognized that Harrod had in mind two kinds of instability, but stuck to the notion that Harrod's work implied a constant capital coefficient: "One can mean two different things by the phrase 'Harrod's knife-edge.' One is the effective-demand instability argument that you elucidate. The other is an existence question: the idea, also to be found sometimes in Harrod, that equilibrium growth need not be possible if n, S, and the capital-output ratio are independently given constants. In 1956, I was mainly discussing this second meaning" (Solow to Nikaido, 29.03.1976; cf. Boianovsky and Hoover 2009, pp. 7-8).

of research at the time, very little of their analysis plays a role in today's thinking." This is an understatement.

Figure 5 shows the relative number of articles in the JSTOR archive that cite "Harrod" and "growth" relative to "Solow" and "growth," starting in 1939. It is easy to spot the high tide of growth economics around 1970. Citations to both Harrod and Solow fall substantially after that. What is less striking, but true nonetheless, is that even as citations to both fall, citations to Solow *relative* to Harrod rise substantially. After 1985, growth-related citations to Solow enjoy a revival, probably related to the rise of endogenous growth models and to the incorporation of the Solow growth model into real-business-cycle models (see Hartley, Hoover, and Salyer 1998, ch. 1). By the end of the period, growth-related citations to Solow outnumber those to Harrod ten to one. Harrod has gone down in the memory of the economics profession as builder of the first modern model of economic growth, although in the eyes of his readers the model with fixed proportions was at best a first step towards an adequate model of growth.

One of the ironies of the history of modern macroeconomics is that real-business-cycle models use Solow's long-run growth model, in which the economy never deviates from the warranted rate of growth, to explain short-run fluctuations. For Solow, this is precisely the opposite of what he sees as Harrod's error: using long-run tools for a short-run problem. Solow however is a short-run Keynesian, who agrees that savings and investment decisions need not be coordinated *ex ante.* To not address this issue in his growth model was an analytical choice, not a claim that such problems do not arise in the world: [26]

[26] As Solow mentions a few years before the publication of his "Contribution": "[S]uppose that people are endowed with perfect foresight as to the future of prices, output, and interest rate. This is always a hard assumption to swallow, and it turns up often in economics. In the present context it is perhaps not so bad. For one thing we are not interested in deducing the implications of any particular method of forming expectations about the future; we might just as well assume the future to be known. Secondly, we are concerned with a long-run equilibrium situation, which is hardly compatible with consistently false expectations" (Solow 1953-154, p. 76).

> It is not my contention that these problems don't exist, nor that they are of no significance in the long run. My purpose was to examine what might be called the tightrope view of economic growth and to see where more flexible assumptions about production would lead a simple model. Under-employment and excess capacity or their opposites can still be attributed to any of the old causes of deficient or excess aggregate demand, but less readily to any deviation from a narrow "balance." [p. 91]

Solow served advanced notice that he was unsympathetic to what, after the 1970s, amounted to the collapsing of short-run into long-run analysis through appeals to rational expectations: "No credible theory of investment can be built on the assumption of perfect foresight and arbitrage over time" (p. 93).

Our contention has been that, however valuable Solow's exploration of flexible assumptions has proved for the theory of long-run economic growth, Harrod was not the right target. His analysis was not of long-run economic growth, but of short-run or medium-run economic dynamics. He made no explicit assumptions about the flexibility, or lack thereof, of the production function, and gave substantial reasons to reject the idea that he thought that a fixed-proportions production function with constant parameters governed the economy. His instability was not Solow's knife edge (i.e., a divergence of the warranted from the natural rate of growth), but rather forces that would drive the actual rate of growth away from the warranted rate. His main concern in the long run was not how the warranted rate of growth adapted to the natural rate of growth, but rather that a warranted rate in excess of the natural rate implied constitutional weakness in the macroeconomy, a the tendency for recessions to be more frequent and more enduring.

It was, perhaps, not inevitable that Harrod was misunderstood, that, in the translation of his ideas into the language of formal model, his key concerns were overlooked and his causal mechanisms were misrepresented. But it was natural. From the point of view of the postwar community of modelers, Solow's interpretation of Harrod's theory was a way of making it

comprehensible – a requirement of the culture of understanding in which they practiced economics. The various aspects of Solow's interpretation of Harrod stem from the academic discourses about dynamics and growth during the time between Harrod's and Solow's publications. We argued that it is possible to even speak of a "culture of misunderstanding" owing to changing dominant practices in the economics profession. As member of different interpretive communities, Solow interpreted Harrod along the lines most congenial to the practices of the emerging community of economic modelers. In an unpublished introduction to Harrod's 1946 lectures Solow mentions himself, that

> Harrod is actually trying to address important issues that later growth theory has neglected or assumed away. So there is food for thought here for a 21st century reader, particularly in the aftermath of the financial crisis and recession of 2008-2010. [Solow 2009, p. 3]

Here Solow acknowledges that Harrod's instability concerns the divergence between the actual and warranted rates of growth. And he goes further:

> This part of Harrod's theory is worth serious thought . . . because it is at least taking up a problem that later growth theory has never solved nor even carefully considered. We know as a matter of fact that modern industrial economies are often off their equilibrium growth paths, sometimes in booms and probably more often in recessions. The source of these deviations usually lies in the behavior of aggregate demand. . . . Recent growth theory, if it considers the question at all, seems to rest entirely on confidence—maybe excessive confidence—in price-guided markets to restore equilibrium growth. But that is generally a poor way to deal with episodes of excess or failing aggregate demand. [Solow 2009, pp. 5-6]

Solow's "Harrod-Domar case" represents a specific reading of Harrod's dynamics that is framed by Solow's research practices, the questions he asked, and the communities he was part of. Contributing to the emerging practices of economic modeling, he integrated Harrod's work into his own framework. It is not that Solow could not have possibly understood Harrod because they were part of different communities. Communities are not closed and monolithic entities; Solow and Harrod were both part of several (intra- and extrascientific) communities, they shared

specific ways of thinking, and were separated by others. However, even though it was part of the Harrod-Domar literature to figure out "what Harrod really meant," these attempts were not a historian's or anthropologist's way of investigating the *meaning* of Harrod's "Essay." Instead of making Harrod's work *plausible*, to use Clifford Geertz's words[27], the authors of the Harrod-Domar models reformulated and transformed Harrod's equations in terms of a model. In these processes of reading and interpretation, essential parts of Harrod's dynamics were lost, and it is in this sense that we speak of Solow's "misreading" of Harrod's work.

Cultures of understanding in economics are not the exception, but the rule. Similar to Solow, who presented Harrod as a growth modeler, textbooks have for decades now told and retold the same success story of growth theory. With modeling becoming the dominant research practice of economists, the history of economics moved to a new path, and the professional identity of economists was transformed. Mainstream textbooks are introductions to the dominant practices of economists, instruments for forging students' identities as modern economists – scientists and modelers – and for steering them away from "unclear" and "unscientific" economics. Economists hardly ever question the potted stories about the development of their field. Their focus is on doing economics, not doing history. The power of such stories nonetheless unfolds on a meta-level: they convey an image of economics as a linear accumulative development of formalization that reinforces the consciousness and self-image of economists. Against the implicit picture of the textbooks and their potted stories, the history of economics – just as the history of any other social and cultural enterprise – is complex and

[27] Geertz (1973) defines culture as a "historically transmitted pattern of meanings embodied in symbols, a system of inherited conceptions expressed in symbolic form" (p. 89). Geertz develops the concept of "thick description" as his method of doing ethnography. With regard to scientific cultures, the basic idea is to make scientific practices plausible in a manner that would allow outsiders to get a grip of the meaning of the participants' procedures and problems.

messy: the creation of stories that cohere with the developing mainstream practices is a process of highlighting and neglecting, valuing and devaluing, translating and passing over what seems to be untranslatable. The gains of a coherent story are offset by the loss of content and a systematic misunderstanding of the past. One job – amongst others – of the historian is to reimagine earlier cultures of understanding, to recover what is lost, to translate what appeared to be untranslatable, and tell coherent stories from now unfamiliar perspectives.

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Figure 1

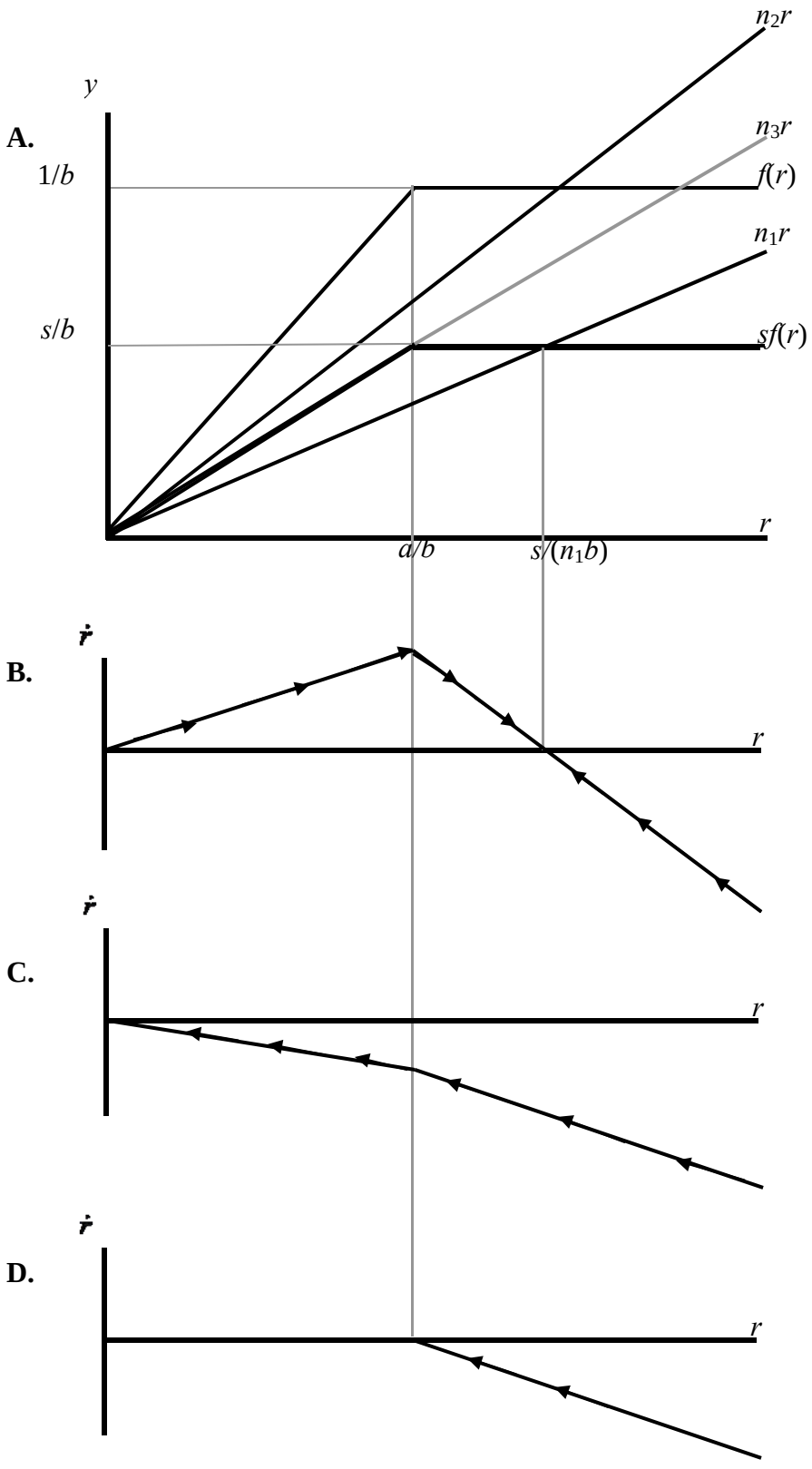


Figure 2

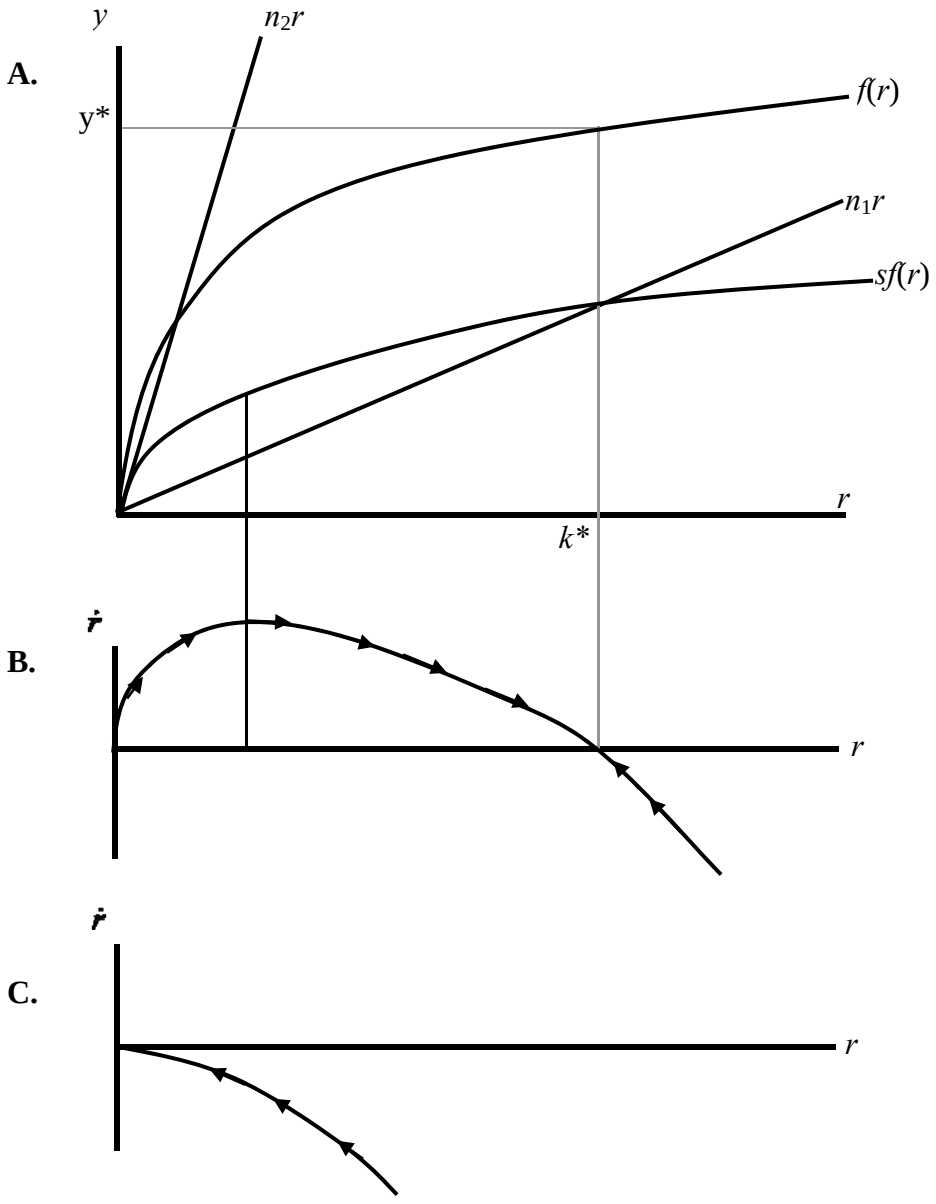


Figure 3

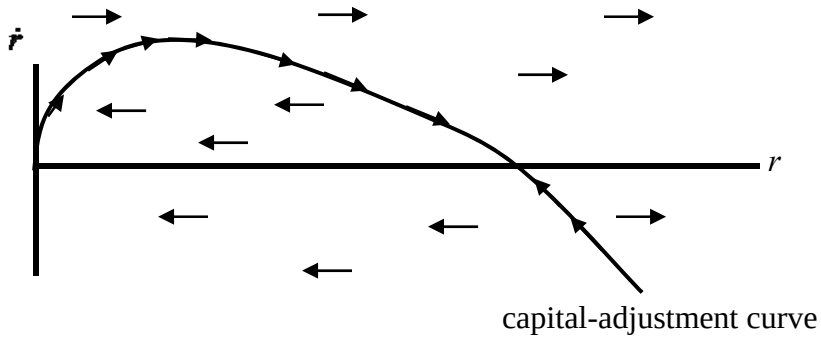
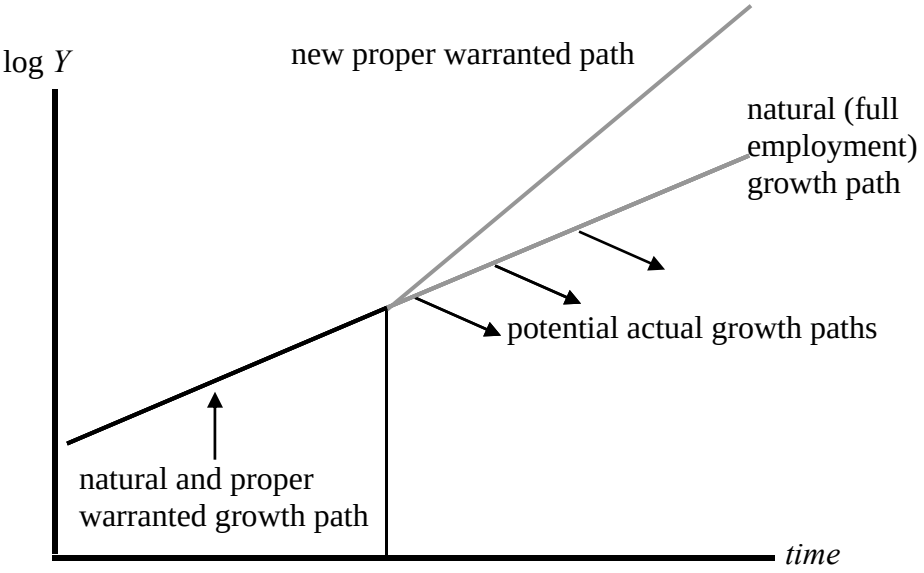


Figure 4

A.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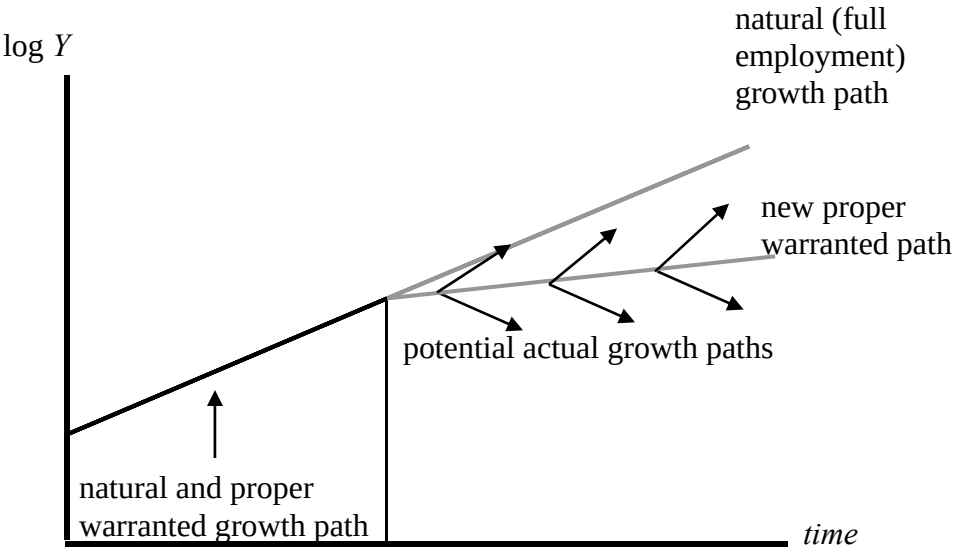
new proper warranted path
natural (full employment) growth path
potential actual growth paths
natural and proper warranted growth path
time

B.


log *Y*
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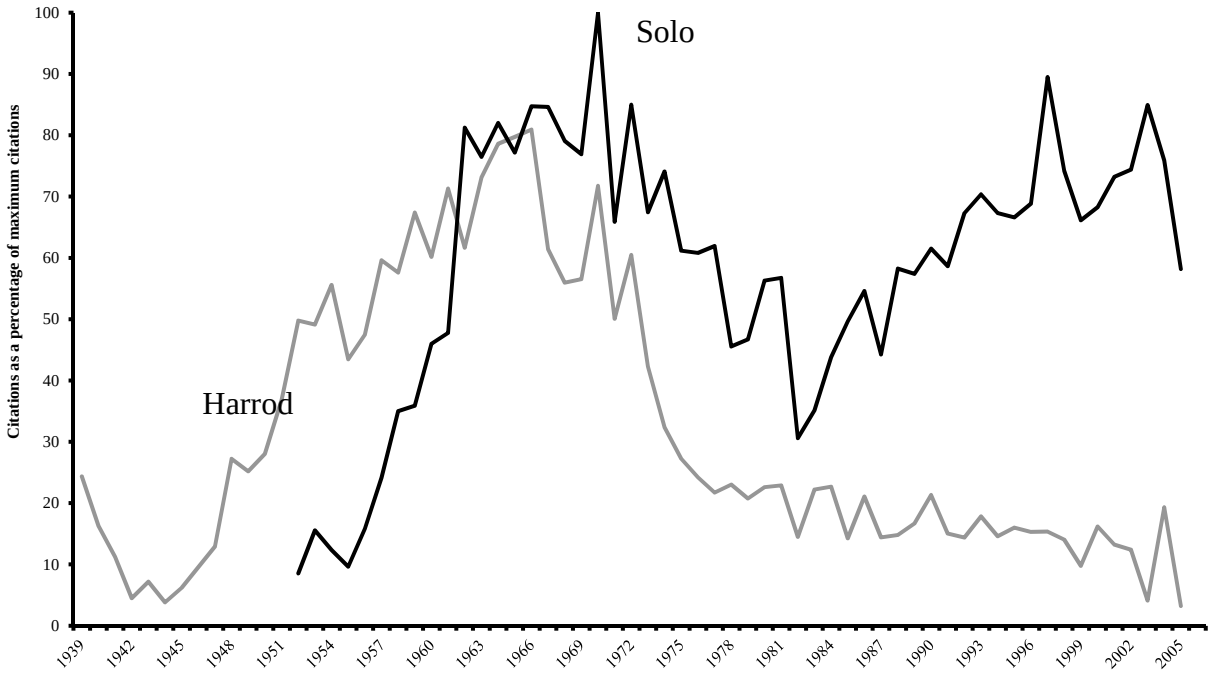
new proper warranted path
potential actual growth paths
natural and proper warranted growth path
time

Figure 5.
Relative Citations of Harrod and Solow on Growth



Notes: Data are the number of articles in economics journals in the JSTOR Archive in which ("Harrod" and "growth") or ("Solow" and "growth"), scaled by the total number of articles and indexed so that the maximum value (177 for Solow in 1970) = 100. Harrod data run 1939-2005; Solow data, 1952-2005. Data were collected from 79 journals in JSTOR on 8 May 2008.