

Economics and Ethics meet under the same umbrella: Edgeworth's 'exact
utilitarianism' (1877-1881)

This paper discusses the intellectual context and motivations that stand in the Background of Francis Edgeworth's mathematical formalization of utilitarianism. Such a project may sound familiar to historians of economics, who recognize it from Edgeworth's 1881 monograph, *Mathematical Psychics (MP)*, one part of which was devoted to the 'economic calculus' and the other to the 'utilitarian calculus'. However, it is significant that Edgeworth was engaged with the second (utilitarian) part long before he started to deal with the first (economical) part¹. Indeed, the whole of the 'utilitarian calculus' had already been published in *Mind* two years earlier, under the title 'The Hedonical Calculus'² ('HC'), while a premature version had even been presented in the concluding parts of his first monograph, the 1877 *New and Old Methods of Ethics*³ (N&O).

Focusing on Edgeworth's motivations in formulating what he ascribed the name of 'exact Utilitarianism' reveals two major tendencies. The first one is his concern to argue in favor of utilitarianism (Henry Sidgwick's version of the doctrine, in particular) and to protect it from its two 'rivals' of the time: 'intuitionism' and 'ethical egoism'. His arguments to this effect were tendency made in the context of the intensive disputes of the time between the three 'camps' in the field of ethics. The second is that the best way to deliver this argument, at least so far as Edgeworth saw it, was to present utilitarianism in the most scientific way (according to the way 'science' was perceived at the time, of course).

Thus, I would like to argue that while Edgeworth's formalization of utilitarianism was not at all the consequence of an awareness of theoretical developments within economics of the time (as opposed to the impression that may arise on a reading of only *MS*), the field of ethics, nevertheless, was under the influence and inspiration of the same trends in science, as was the field of economics. This insight has some significance for the historians of economics who in recent years, have devoted much attention to exploring the ways in which economists of this time (Jevons in particular) were inspired by the natural sciences.⁴

¹ This was also noticed by Creedy (1986) Uemiya (2008) and Barbé (2010).

² Edgeworth (1879) the 1881 version included also appendixes.

³ Edgeworth (1877)

⁴ Prominent examples would be: Mirowski (1989), Schabas (1990), Schabas (2005), White (2004), Maas (2005).

Now, in addition to the attempt to recapture Edgeworth's motivations in his formalization of utilitarianism, it will also be a concern of this paper to deal with the implications of Edgeworth's decision, following his acquaintance with Jevons's economics, to put the two 'moral sciences', economics and utilitarianism, together under the one umbrella of "Mathematical psychics". This decision, so I argue, locates this book from retrospectively as an illustration of a very intriguing phase in the history of the relation between utilitarianism and economics; a phase in which utilitarianism was not only considered as a meta-criterion for economic policy but was addressed in new ways, not necessarily the most applicable ways, but highly theoretical, and translated into the language of formal mathematics. This phase/episode reflects economists' lack of satisfaction with general estimations of aggregate utility alone, as opposed to their predecessors, and as a consequence a turning up-side-down of the traditional relations between economics and utilitarianism: addressing utilitarianism as a branch of 'economics',⁵ and not (only) the other way around.

Thus, the first and the last sections of the paper will locate *MS* in the context of the relations between economics and utilitarianism, while the inner sections will attempt to recapture Edgeworth's motivations in his project (ending with his 1881 work).

1) Mill and Jevons: keeping utilitarianism and mathematics apart

From the point of view of English economists in the second half of the 19th century, it seems almost inevitable that two of their greatest (albeit disputed) inspirations - on the one hand, professionalization and mathematization, and on the other, utilitarianism - would be somehow theoretically unified. Then again, it had not occurred in previous generations. Interestingly enough, this characteristic of earlier scholars to hold the two apart was shared by the two prominent yet very distinct economists (and utilitarians), J.S. Mill and W.S. Jevons.

Both Mill and Jevons considered themselves as faithful utilitarians but obviously they each offered very distinct interpretation of the doctrine.⁶ Indeed, the divergence

⁵ 'Economics' in a later sense, such as the later 'welfare economics' or 'social choice' theories, but back then it was addressed as 'mathematical ethics'.

⁶ While Mill presented a modified and detailed version of the moral doctrine in his celebrated *Utilitarianism* (1861), it was Jevons who made it perfectly clear in his later article in the *Contemporary Review* (1879) that he (not a philosopher, yet a scientist interested in the subject) does not accept Mill's interpretation and consider himself as a better follower of Bentham's idea. Thus, Jevons rejected Mill's differentiation between kinds of pleasures on a qualitative base, addressing it as a

between their interpretations might lead to the following question: what makes them both utilitarians? Furthermore, how are we to understand the doctrine in our current discussion? For present purposes, I would suggest 'utilitarianism' be understood as an acceptance of two postulates: (i) that 'Social utility' is an aggregate (estimated or precise) of individual utilities (reflecting individuals' pleasure, happiness etc); (ii) that making social utility as great as possible is the ultimate moral imperative (for an individual/ a governor). While each accepting both of those basic utilitarian principles, Mill and Jevons indeed were very much divided in their different interpretations and implementations of the doctrine. In the narrow context of this paper, however, our main concern is the way in which the two scholars viewed the relations between utilitarianism and their economic theories. As far as this question is concerned it seems that the two moral scientists had a lot in common.

Thus, both of them considered utilitarianism as highly relevant for economic practical policy decision making (in a general estimation manner).⁷ Policy decisions, however, were not considered by them as a scientific task, but as a general estimation, a part of the field of policy making, open for disputes and discussions.⁸ In addition, neither of them considered utilitarianism to be a precise doctrine, nor did either of them formulate it mathematically, i.e. (what we would address) as a branch of economics.⁹ From an economic-theoretical point of view, as a part of the classical tradition (and having 'sensitivities' and considerations of his own¹⁰) Mill avoided using the concept of utility in a quantitative-mathematical context in his economic science.¹¹

paternalistic approach as well as a more 'genial' but less progressive and up-to-date one. See Jevons (1879)

⁷ S. Peart has shown how Jevons was at one with Mill in using utilitarian evaluations, together with the criterion of promoting liberty, in order to assess and justify a wide range of policy issues. Peart (1996) pp. 152-169

⁸ "We cannot expect to agree in utilitarian estimates, at least without much debate. We must agree to differ, and though we are bound to argue fearlessly, it should be with the consciousness that there is room for wide and bona fide difference of opinion." Ibid. p. 166

⁹ Utilitarianism was considered by Mill as a meta-justification for policy as a whole, but only a general-estimation compass. This is how one should understand for instance Mill's comment in a letter: that he was but a part of a chain of political economists that also included Smith, Turgot, Say, Ricardo and James Mill, "none of whom regarded political economy as anything but a subordinate though necessary branch of utility..." Letter to Lalor, 27 June 1852

¹⁰ See De Marchi 1973; Hollander 1985, 934-936; Hollander 2000, chapter 9

Also see my account of the subject in: "The 'Utility' and Economics of John Stuart Mill: A Reexamination", paper submitted to HOPE

¹¹ Thus, in Mill's economics, utility does not play a central role in the theory of value, nor does it serve for an account of consumer choice, satiation, or pleasure-pain tradeoffs. It is simply not linked to the scientific relation of quantitative causality in any basic sense.

As for Jevons, although he does suggest the possibility of a "higher calculus of moral right and wrong" he did not engage in it.¹² While formulating a mathematical representation of the individual utility (i.e. the 'lower feelings') and basing his whole theory on it,¹³ he nevertheless did not use it as a platform for a mathematical formulation of social aggregate utility. As he was very explicit about: "The reader will find, again, that there is never, in any single instance, an attempt made to compare the amount of feeling in one mind with that in another. I see no means by which such comparison can be accomplished"... "Every mind is thus inscrutable to every other mind, and no common denominator of feeling seems to be possible".¹⁴ The consequence of which is the inability to sum up utilities.

2) Edgeworth's 'Exact Utilitarianism' in philosophical context

It was Jevons's friend, Edgeworth, who will not hesitate to combine mathematics and utilitarianism in the most celebrated and explicit manner, and in addition will integrate in some cases a utilitarian apparatus inside his economic models (i.e. as a part of the individuals' motives while negotiating an exchange, as will be shown). Edgeworth, born in 1845 to an Irish family, was tutored as a child in classics and mathematics, and after four years at Trinity College, Dublin, he entered Oxford where he took first class at the Final Examination in *Literae Humaniores* (1867-1869). The following (for our story) critical years, he spent in London. There he studied on his own mathematics, law and philosophy, living on a modest private income (and later, from 1880, lecturing on Logic and Political Economy at Kings College)¹⁵. His initial knowledge in economics was acquired only in 1879-1880 with the guidance of his neighbor in Hampstead – Jevons. In 1879 he apparently read for the first time Jevons's *Theory of Political Economy* (the second edition was published in that year) as well as

¹² As was mentioned above, the utilitarian imperative was considered by utilitarians as a normative guidance not only for rulers, but also for moral individuals. Jevons calls this imperative "the higher motive" but he famously separated this acknowledged motive from the scientific-economic discussion: "I have no hesitation in accepting the Utilitarian theory of morals which does uphold the effect upon the happiness of mankind as the criterion of what is right and wrong. But... "My present purpose is accomplished in pointing out this hierarchy of feeling, and assigning a proper place to the pleasures and pains with which the Economist deals. It is the lowest rank of feelings which we here treat". Jevons (1888) [1871] "Relation of Economics to Ethics" paragraphs 29,35

¹³ "My principal work now lies in tracing out the exact nature and conditions of utility. It seems strange indeed that economists have not bestowed more minute attention on a subject which doubtless furnishes the true key to the problem of Economics" Jevons (1888) Chapter 3 paragraph 12

¹⁴ *ibid.* paragraph 20

¹⁵ Keynes (1963) [1933] pp. 223-224, Newman (1987) p. 85, Baccini (2011) pp. 234-235

Marshall's *Pure Theory of Domestic Value*.¹⁶ The new approach to economics embedded in those books had a massive impact on Edgeworth's economics, and in his *Mathematical Psychics* (1881) he highlighted his and Jevons's shared views as to the crucial role of individuals' utilities as "the true key to the problem of Economics"; in addition he expressed therein a similar adherence as his friend to the centrality of the differential calculus as the main tool to address problems in economics.¹⁷ In contrast to Jevons, however, he used extensively the same tool as the formal language to address questions in the aggregate level of utility (using integral functions). But that engagement was the product of a separated earlier story, or so it seems.

While Jevons was for Edgeworth the model for an *economist* - the one who had placed 'utility' at the center of economic analysis, and used the calculus extensively, it was Henry Sidgwick who was for him the greatest *utilitarian*. Published in 1874, Sidgwick's *Methods of Ethics* had an enormous impact on Edgeworth, and apparently transformed him into a follower of Sidgwick's version of the doctrine. Hence, Edgeworth directed the readers of his *O&N* (1877) and 'HC' (1879) to take his theory of Utilitarianism to be a mathematical interpretation *consistent* with Sidgwick's philosophical account (equipped with the tools of the cutting edge scientists of the period). In his *O&N* Edgeworth titled that interpretation as "exact Utilitarianism":

The doctrine of Fechner and Sidgwick may be termed exact utilitarianism, as distinguished from Hume's non-quantitative principle of utility, and the not very explicit greatest-happiness principle of Bentham and his followers, including J.S. Mill.¹⁸

That paragraph is located in a critical place in the monograph: where the philosophical (and verbal) discussion ends and the mathematical discussion begin. The book up until this point is nothing but a re-justification of Sidgwick's philosophy, established by way of comparing it with the alternative doctrines of ethics. Sidgwick, the Cambridge philosopher (a purely *academic* philosopher, as opposed to previous prominent utilitarians such as Bentham and Mill), presented in his *Methods of Ethics* a

¹⁶ Howey (1960) p. 101 See also Edgeworth's comment in *MS* that while writing the *Mind* article (1879) he was not aware of Jevons's ideas of the subject.

¹⁷ Famously, this adherence to the differential calculus was heavily inspired by the mathematical physics of the period. See Mirowski (1989) pp. 217-222. Edgeworth himself couldn't be more explicit about the many poetical analogies to the natural sciences in the book. Note that the title of the book is itself an allusion to 'Mathematical Physics'. A prominent figure for him in this respect was the Irish physicist, William Rowan Hamilton (a friend of the family; see Newman p. 87).

¹⁸ Edgeworth (1877) p. 35

meticulous analysis of the three basic competing doctrines of ethics: 'egoistic hedonism', 'intuitionism' and 'utilitarianism';¹⁹ examined the rationality of each one of them as an ethical guidance (mostly for individuals), distinguished carefully between them,²⁰ and ultimately argued for the superiority of utilitarianism over the other two, although he *did not* dismiss the other two, nor adhere to 'pure utilitarianism' on the practical level (both for individuals and for central planners).²¹ Edgeworth totally accepted Sidgwick's analysis and expressed his admiration for him in many places in his *N&O*.²²

However, Edgeworth was also well aware of the lively controversy that Sidgwick's book aroused. Much of the controversy was argued through the new journal (founded in 1876) - *Mind*.²³ Intuitionists from the one side, ethical-egoists from the other, reviewed and criticized its contents. It is crucial to notice how this dispute, ethical in its basis ('moral reasoning'), had some strong linkage with other key questions in philosophy of the time, and especially to the question of the motives of the will ('practical reasoning' and 'free will').²⁴ Generally speaking one can argue that the strength of the egoist/reductionist side of the argument was that it was very well connected with the sciences of the time (as will be explained), while the strength of the intuitivist side was that of protecting the uniqueness of humans and the concept of free will. It seems that Edgeworth aimed at both of these advantages.

One critic that caught Edgeworth's attention in particular, was Alfred Barratt's,²⁵ as shown in the chapters discussing the two approaches (attempting to reconcile them,

¹⁹ "By Utilitarianism is here meant the ethical theory, that the conduct which, under any given circumstances, is objectively right, is that which will produce the greatest amount of happiness on the whole; that is, taking into account all whose happiness is affected by the conduct. It would tend to clearness if we might call this principle, and the method based upon it, by some such name as "Universalistic Hedonism". Sidgwick (1962) p. 411

²⁰ While the distinction between utilitarianism and intuitionism was generally acknowledged (based primarily on *a priori* versus *a posteriori*/consequential reasoning) it was a harder task, or so it seems, to make people deeply internalize the distinction between ethical egoism and utilitarianism. Both J.S. Mill and Henry Sidgwick made a lot of effort in this direction. See Mill (2000) chapter 3, Sidgwick (1962) pp. 411-412

²¹ See Sidgwick (1962) Book IV chapters 1,2. Schneewind chapter 6

²² Edgeworth mentions Sidgwick 28 times in his short monograph, always in a highly positive manner and at times as an admired figure (see instances in Edgeworth 1877 pp. 11, 13, 14, 32, 60, 68).

²³ *Mind* was founded by Alexander Bain (see footnote 43) and its first editor was George Croom Robertson. In its first years the Journal put emphasis on connections between philosophy and the new psychology of the time.

²⁴ Sidgwick did try to disconnect the two subjects, the descriptive and the normative. See Sidgwick (1962) p. 412

²⁵ As Edgeworth's full title of his 1877 book suggests: *New and old methods of ethics: or "Physical ethics" and "Methods of ethics"*. (The former refers to Barrat's book, the later to Sidgwick's).

referring to Sidgwick's as superior, ultimately²⁶). In 1877-8 Barratt expressed his later objections to utilitarianism, and his adherence to 'ethical egoism', publishing a series of papers in *Mind* criticizing Sidgwick's *Methods*. Barratt addressed the *Methods* as an obsolete kind of ethics, based on the old fashioned method of introspection rather than on scientific methods alone.²⁷

Barratt, a brilliant scholar, might have been familiar to Edgeworth from his years at Oxford; about the same age, an outstanding scholar,²⁸ and sharing the same fields of interest. Edgeworth was still at Oxford when Barratt's *Physical Ethics* was published in 1869. This book reflected Barratt's early aspiration to provide a scientific formulation of ethics. His view of 'scientific ethics' included, first (an approach that Sidgwick will share) a separation between 'pure ethics' and 'applied ethics', with a declaration to focus on the 'pure' aspect,²⁹ second, a hope to connect pure ethics with natural sciences of the period such as physics and psychology ('as one of their branches'),³⁰ third, severe skepticism with regard to any traditional views of ethics.³¹ The style of writing was in itself an expression of his scientific tendency, composed of Euclidean-style axioms, definitions and propositions. In Barratt's view when one adopts these methods one cannot but justify a 'physics of ethics' based on the leaning of individuals to actions that have pleasures as their ends. In other words, he argues for a psychological egoism which, for him, must have concluded in ethical egoism. Obviously he also rejected utilitarianism on these grounds.³² His later criticism on Sidgwick was thus a natural continuation of his earlier tendencies.

As we shall see, Edgeworth in his three works could avoid this kind of criticism because, together with a total acceptance of Sidgwick's account of ethics, he also presented it with the tools of mathematics (the same kind used by the most up-to-date

²⁶ Uemiya (2008) presents Edgeworth's philosophy as an outcome of the Sidgwick-Barratt controversy, but I disagree with his 'symmetrical' interpretation, which ascribes the two scholars an equal weight as influences on Edgeworth.

²⁷ Barratt (1877), Barratt (1878)

²⁸ In 1866 Barratt was achieving the unequalled distinction of five first classes from beginning residence (Balliol college). He obtained a fellowship at Brasenose a year later, and in January 1869 he published his 'Physical Ethics,' with which he had 'amused himself' in leisure hours at Oxford (he passed away in 1881, at the age of 37, attacked by paralysis). From the Dictionary of National Biography, 1885-1900, Volume 03 (written by Lesley Stephen)

²⁹ Barratt (1869) p. 1

³⁰ "In the appendices I have attempt... to explain in an imperfectly connected series the relations of the moral system upheld in the text to those wider principles of mental and physical science with which as one of their branches it must necessarily come into contact." Ibid. p. iv; see also analogies and attempts to connect ethics to science for instances pp. 4, 25

³¹ Ibid. p. 3

³² Ibid. pp. 70-75, 252-254

natural and social scientists, no less); moreover, his ethics was connected to psychology in a stronger way than was Barratt's. What could Barratt, who did not use mathematics at all in his 'physical ethics', and only loosely connected it to the natural sciences, complain about?³³

Thus what Edgeworth had to offer in his *N&O* was similar to Sidgwick: an Ethical stand in the middle between 'ethical egoism' and 'intuitivism', which does not deny the other two kinds of motives, but yet siding in favor of the superiority of utilitarianism as a normative imperative. The more scientific character of his 'exact utilitarianism', however, helped him to present the same doctrine in a more modern fashion and in so doing to improve the philosophical argument. To the intuitivist he would be able to say (in addition to Sidgwick's philosophical arguments): your doctrine is not enough in accordance with the science of the age; to the ethical egoist he would answer: my doctrine is not only more subtle and human, but it is also *no less* scientific than yours.

3) The Scientification of Utilitarianism

Interestingly enough, the path to ascribing ethics with its *new* scientific character entailed passing through and connecting it with three scientific trends of the period: 'psychophysics', Mathematical formalizations, and theories of evolution (the first two are in common with the developments in economics of the same period, Jevons's in particular).

With regard to 'psychophysics', ideas of a unified science of physical and mental phenomena where 'in the air' and of greatly attracted many philosophers and economists.³⁴ Note that Edgeworth and Jevons were apparently independently influenced by this trend. While Jevons's inspirations in this respect, already presented in the first edition of his *Theory*, were mostly the British mental scientists Bain³⁵ and

³³ And indeed Barratt's view of Edgeworth's work was much more sympathetic than his view of Sidgwick's; see: Barratt (1878).

³⁴ See Schabas (2005) pp.134-136 and Maas (2005) 159-169. Schabas located the emergence of economists' growing interest in psychology with Mill's version of associationism as a preliminary stage to 'psychophysics' (Mill referred to Bain, see footnote 35). Maas put more emphasis on the essential differences between Mill's approach to psychology and later economists such as Jevons's adherence to the subject.

³⁵ Alexander Bain (1818-1903), Scottish in his origins, was a highly influential scholar of 'mental philosophy' ('psychology' in our words). Bain combined associationist psychology with an analogy between the mental and the physical phenomena that allowed for the employment of measurement procedures in researching mental phenomena. His main two works: *Senses and the Intellect* (1855) and *Emotions and the will* (1859) attempt to reconcile the associationistic concept of the mind and the new ideas, while protecting the concept of the freedom of will, i.e. Bain did not accept an extreme reduction of the mental to the physical. (Note that Barratt referred to Bain extensively in his *Physical Ethics*, not always as a supporter).

Jennings³⁶, Edgeworth's inspirations were not only English, but also included the German psychophysics of Wundt, Helmholtz and Fechner (thus more up to date). Edgeworth had the best mediator to access those developments in science: his neighbor and friend, James Sully, who helped him with his book.³⁷ Sully, who would subsequently become one of the founders of the Psychological Society in England (1901), and one of the first to write English textbooks in psychology,³⁸ was at this point thirties and recently returned from philosophical studies in Göttingen and Berlin, where one of his teachers was Helmholtz.³⁹ Sully himself referred many times in his own *Sensation and Intuition* (1874) to the experiments and works of Gustav Theodor Fechner, the German pioneer experimental psychologist who is considered the founder of psychophysics.

Returning now to the key paragraph from *N&O*, in which Edgeworth turns from detailed philosophical discussions and commences the presentation of his mathematical interpretations, we may observe that it was at this particular point that he connected his philosophical mentor (whose ethics had been discussed throughout the book up to this point) with the cutting-edge scientist; and it is precisely the bringing of the two of them together that generates his 'exact utilitarianism', i.e. the "doctrine of Fechner and Sidgwick."⁴⁰

What Fechner and Wundt had to offer the utilitarian was a programme of psychophysics based on experimentation (as opposed to the method of introspection of the associationist psychology), within which the effects of a physical stimulus on individual's sensations (pleasures or pains) were expressed as a mathematical function (reflecting the tendency of the effect of a stimulus at some point to *decrease* with every sequential pulse).⁴¹ An *expansion* of that formalization from an individual to a group of individuals could generate a new expression for the use of utilitarianism, and

³⁶ On Richard Jennings's inspiration on Jevons, through his *Natural Elements of Political Economy* (1855) see White (1994)

³⁷ Edgeworth devoted the acknowledgment in the book to Sully alone: "I have to thank my friend Mr. James Sully, author of "Sensation and Intuition", "Pessimism" &c., for having revised and corrected the following pages during their passage through the press, and for many suggestions." Edgeworth (1877)

³⁸ Valentine, E. (2001) p. 405

³⁹ On Helmholtz see Cahan (1993) p. 1.

⁴⁰ Edgeworth (1877) p. 35

⁴¹ That feature economists associated with diminishing 'marginal utility'; 'final degree of utility' in Jevons's words. Note that Edgeworth used that principle as a *postulate* basing it on 'every-day experience', recognized by Laplace, and confirmed by the new formulations of Fechner and Wundt (apparently disconnected with Jevons's formulation). Edgeworth (1877) p. 41, 69 Edgeworth (1879) p. 397

thereby provide 'a service' for the new utilitarian.⁴² But it is important not to confuse this method with the ethical imperative (which is not a part of the psychophysics agenda, of course). Edgeworth's utilitarianism, as Sidgwick's, does not exclude sensual pleasures as motives in effect, *but* insists that the ethical apparatus be the maximization of the aggregation of those effects.

Thus, in *N&O* as in his later 'HC' Edgeworth presents abstract problems of allocation (of stimulus⁴³, and then of 'means to stimulus'⁴⁴ such as wealth and labor⁴⁵) so that the aggregate of pleasures/happiness, the result of that allocation, will be at a maximum. Each and every problem entails few possible mathematical interpretations, and the author selects his preferred interpretation based on the philosophical and ethical nuances that it captures (i.e. the one that fits the most with his idea of exact utilitarianism).

Edgeworth's preferred kind of mathematics to deal with these theoretical questions, due to the properties of the basic functions of pleasure-producing,⁴⁶ and due to the posing of the problem as a minimum-maximum one, is 'the calculus of variations'; in which the aggregates are formulated as integrals.⁴⁷ Edgeworth referred to the English mathematician Isaac Todhunter as an authority in the field of calculus⁴⁸ (as opposed to Jevons who referred in his works to De Morgan, Todhunter's teacher). It was probably not a coincidence that it was the same mathematics which was being used by physicists of the time, and was considered as the most trendy and scientific one (See also footnote 17). Taking the argument one step further, albeit it in a speculative manner: it might well be the case that the popularity of this particular kind of

⁴² See Edgeworth's references to Fechner and Wundt: Edgeworth (1877) pp. 20, 35, 40-42, 61, 72, 75 and Edgeworth (1879) pp. 396, 397, 399, 404 [The same as Edgeworth (1881) pp. 60, 62, 65, 75] "This "moral arithmetic" is perhaps to be supplemented by a moral differential calculus, the Fechnerian method applied to pleasures in general. For Wundt has shown that sensuous pleasures may thereby be measured..." Edgeworth (1879) p. 396

⁴³ Two basic problems were presented in *N&O*. 'Problem I': "Given a certain quantity of stimulus to be distributed among a given set of sentients (with the condition that every element is to have *some* stimulus), to find the law of distribution productive of the greatest quantity of pleasure." *Problem II* will be the same not including the condition. Edgeworth (1877) pp. 43-44

⁴⁴ See the rewording of the same two problems with terms of wealth in *N&O*, *ibid* pp. 54-55.

⁴⁵ The basic question presented in 'HC' is: "To find the distribution of means and of labour, the quality and number of population, so that there may be the greatest possible happiness." Edgeworth (1879) p. 394

⁴⁶ A central exemplar mentioned by Edgeworth is Wundt's curve of pleasure of which: "the higher part is concave, the lower part is convex". Edgeworth (1877) pp. 41, 69

⁴⁷ See the many presentations of mathematic integrals in both works. Edgeworth (1877) pp. 38, 44-53, 58-59, 67. Edgeworth (1879) pp. 394, 399, 400, 402, 403.

⁴⁸ The works Edgeworth referred to are Todhunter's: *History of the Calculus of Variations* (1861) and *Researches in the Calculus of Variations* (1871)

mathematics encouraged Edgeworth to engage with 'exact utilitarianism' in the first place.

A prominent feature of Edgeworth's exact utilitarianism is the interpretation it embodies of the different 'capacities' of individuals (and groups) to acquire pleasure.

In 'HC' he defines the concept of capacity among a set of definitions he provides:

An individual has greater *capacity for happiness* than another, when for the same amount whatsoever of means he obtains a greater amount of pleasure, *and also* for the same increment (to the same amount) whatsoever of means a greater increment of pleasure.⁴⁹

The implications of this presumption, as Edgeworth explicitly suggests, is the separation of *exact* utilitarianism from equality, the formal omission of '...for the greatest number' from the utilitarian apparatus. It may be better to have a few strong lamps than many weak ones when you have limited resources and maximum light as your goal.⁵⁰

The very idea of difference in 'capacities' is inspired by evolutionary theories. It was Herbert Spencer's version of evolution in particular that had the greatest influence on Edgeworth.⁵¹ In *N&O*, when discussing the possibility of ascribing different capacities within exact utilitarianism, he quotes Spencer (from the first edition of *Mind*):

...the higher the evolution rises the stronger do the emotions become. For as the increasingly-complex emotions successively developed result from integration of pre-existing groups of actual and nascent sensations, the resulting totals must grow continually larger, &c."

Edgeworth continues: "With greater quantity may, perhaps, be enumerated more pleasurable quality, more sympathy, easthetic feelings, sense of dignity⁵²..."

⁴⁹ Edgeworth (1879) p. 395; in addition, the different capacities in acquiring pleasures is paralleled with different capacities of suffer labor.

⁵⁰ Edgeworth (1877) p. 74, Edgeworth (1881) p. 117

⁵¹ Contrary to Peart and Levy's interpretation according to which Edgeworth was a reconciler of Darwin and utilitarianism. Levy and Peart first offer a distinction between Spencer's and Darwin's approaches to the question of equality, and then place Edgeworth in Darwin's 'camp' (with the 'bad guys'); see: Levy and Peart (2005) pp. 208-233. Nevertheless, the two do not address the fact that Edgeworth actually did not refer at all to Darwin (or to biology) in his earlier works, in which he developed his ideas, and only mentioned him in the later *MS* appendixes. Moreover, it is no wonder that he leaned heavily on Spencer, since the latter served for him as an example of a reconciler of utilitarianism, social evolutionism and the new insights of psychology. Naturally, Edgeworth saw himself as continuing this legacy.

⁵² Edgeworth (1877) p. 72

Within his works discussed here Edgeworth aspired to provide a mathematical interpretation of these insights, representing individual's 'capacities' and (privileged) future generation's utilities inside the integrals.⁵³ By that he demonstrated originality in coping with the challenge of reconciling the *static* nature of utilitarianism with the *dynamic* character of evolutionism, a challenge that intrigued and troubled scholars of the period (as reflected in debates in *Mind*).⁵⁴

One important feature of 'exact utilitarianism' should be emphasized though. The "exact" part of the concept not only represents a highly scientific version of the doctrine, it also stands for the purely theoretical aspect of it. The intention is to capture the idea of maximizing aggregate happiness in its most purified formulation, not at all to provide a sufficient practical guide. Thus, for example, Edgeworth comment:

Nothing indeed appears to be certain from a quite abstract point of view"... "It may be admitted however that a limit below the zero of happiness, even if abstractedly desirable, would not be humanly attainable;...Let Politics and Political Economy fix some such limit above zero."⁵⁵

This important distinction between purely theoretical versus practical interpretations of utilitarianism is crucial in particular in order to understand how it was that despite of his departure from the maxima of equality, Edgeworth still saw himself as a loyal interpreter of Sidgwick's ethics⁵⁶ (and as developing Spencer's legacy⁵⁷). What Edgeworth have found attractive in particular in Sidgwick's account is that while on the practical level the utilitarian apparatus of aggregation was considered by him as a 'rough' estimate combined with other considerations,⁵⁸ it could also be based, however, on as accurate as possible a *theoretical* account; and for this purpose it was recommended by Sidgwick to ascribe as much 'exactness' to the estimations of

⁵³ See Edgeworth (1877) p. 67-68 Edgeworth (1879) p. 400

⁵⁴ A previous attempt to reconcile the two was Spencer's (using a different concept of utilitarianism i.e. an 'indirect' one. within this account evolution brings about an increasing amount of happiness, and by that resolve itself into Utilitarianism) see: Weinstein (1998) See also Jevons (1879) and Sidgwick's rejection to the using of evolution as a part of the account of ethics: Sidgwick (1876)

⁵⁵ This is a case were the distribution of wealth is at a maximum "when the lower classes sacrificed to that of the higher classes. And again the happiness of part of the second generation may be sacrificed to that of the succeeding generations." Edgeworth (1879) p. 404

⁵⁶ As opposed to Uemiya's view of the subject; see Uemiya (2008) p. 16-17

⁵⁷ As opposed to Peart and Levy's interpretation; see footnote 51

⁵⁸ Exact estimations were possible only to a limited extent in Sidgwick's view, and addressing the question of distribution, for example, required some other values *outside* of utilitarianism to supplement it (such as 'just' or 'right'); Sidgwick *ibid.* p. 416

aggregate utility as possible.⁵⁹ This message, although not a central one in Sidgwick's book, fit together with his own attempt to present the *'Methods of Ethics'* as a scientific effort,⁶⁰ "an attempt to introduce precision of thought into a subject usually treated in a too loose and popular way".⁶¹ Obviously, his was a completely different understanding as to what would be a scientific exploration of the field.⁶² Nevertheless, this was sufficient for Edgeworth to consider himself as the mathematically better-equipped follower of the philosopher.

4) *Mathematical Psychics*: bringing economics and ethics together

The two years following the publication of 'HC' in *Mind* are quite dramatic ones in our story. These are the years in which Edgeworth delved into the new developments in economic theory, and no doubt was enthusiastic, not only as to the genius of the ideas, but also as to the similarities and connections between his approach to ethics, and the economists' new approach to their discipline (both being inspired by the same trends in science). This is probably what led to the decision to put the two subjects of research into one monograph under the one title: *Mathematical Psychics: an essay on the application of mathematics to the moral sciences*.

As it was presented in *MS*, 'the calculus of feelings/ Pleasures' is subdivided into two fields (in a symmetrical manner):

namely, Economics and Utilitarian Ethics. The economical Calculus investigates the equilibrium of a system of hedonic forces each tending to maximum individual utility; the Utilitarian Calculus, the equilibrium of a system in which each and all tend to maximum universal utility. The motives of the two species of agents correspond with Mr. Sidgwick's Egoistic and Universalistic Hedonism'.⁶³

⁵⁹ Sidgwick writes: "...its show of exactness is grotesquely incongruous with our consciousness of the inevitable inexactness of all such calculations in actual practice. But, that our practical Utilitarian reasoning must necessarily be rough, is no reason for not making them as accurate as the case admits; and we shall be more likely to succeed in this if we keep before our mind as distinctly as possible the strict type of the calculation that we should have to make, if all the relevant considerations could be estimated with mathematical precision." Sidgwick *ibid.* p. 416

⁶⁰ *Ibid.* Book I chapter I. See also Schneewind (1986) chapter 6 p. 191

⁶¹ (*Mem* p. 295) quoted in Schneewind p. 291

⁶² Thus, Sidgwick's project was ultimately about providing a systematic, objective, nuanced and comprehensive philosophical research into the alternative methods of ethics, and was not concerned with mathematization of the field whatsoever.

⁶³ Edgeworth (1881) p. 15-16

The part in the book titled 'Utilitarian Calculus' (pp. 56-82) is actually 'HC' in its entirety, as briefly noted by Edgeworth,⁶⁴ while the only changes (traced by Barbé⁶⁵) are the title, and two additional footnotes in which he mentions Jevons and Marshall.⁶⁶

The 'Economic Calculus' includes some genuine development of the theory inspired by the works of political economists such as Marshall, Walras, Cournot, and of course Jevons, who receives the most praise. It was Jevons who probably introduced him to the rest.⁶⁷ The personal acquaintance between Edgeworth and Jevons was probably mediated through their mutual friend (and neighbor in Hampstead), Sully. Sully met Jevons around 1879 and later that year introduced him to Edgeworth. Note that the three were also members of the same London clubs.⁶⁸

The original contributions of Edgeworth to the science of economics, 'the economic calculus', have been broadly discussed in the literature.⁶⁹ The economic devices presented there, such as the indifference curves, the contract curve and the graphical representation that led to the 'Edgeworth box', are all still used in economic courses today. In the context of this paper, one aspect of Edgeworth's economics is of particular interest,⁷⁰ (and this is not discussed in today's classrooms): in the concluding part of 'the economic calculus' a general case is presented in which under the circumstances of imperfect competition the barter between two (or few) agents⁷¹, will not result in one point, i.e, in one relative price of the two commodities, but in set of points (inside the 'core' and on the contract curve). This is the famous 'indeterminacy' of a contract.⁷² (See the illustration given in the Appendix below).

⁶⁴ "The Economical thus leads up to the Utilitarian species of Hedonics; some studies in which already published (under the title of 'Hedonical Calculus' — the species being designated by the generic title) are reprinted here by the kind permission of the Editor of Mind." Ibid p. vii.

⁶⁵ Barbé (2010) p. 87.

⁶⁶ These footnotes provide another indication that Edgeworth was not familiar with Jevons and Marshall's writings before 1879.

⁶⁷ Ibid. p. 92.

⁶⁸ Ibid. pp. 87-89 The two clubs are 'Savile' and 'Athenaeum'.

⁶⁹ See Creedy (1986) chapters 3,4,6 Blaug kjhkhkjh

⁷⁰ See other discussions of this idea in: Creedy (1984), Creedy (1986) pp. 80-81, Barbé (2010) pp. 98-99, Uemiya (2008) p. 14.

⁷¹ or groups of agents: ...٨٢٢١٧

⁷² In Edgeworth's theory as opposed to Jevons's this case warranted a theoretical account. As pointed out by Creedy (1984) p. 614, Jevons mentioned (under the subtitle 'Failure of the Equations of Exchange') exceptional cases of 'indeterminate bargains', that should be resolved somehow; See Jevons (1888) [1871] chapter IV paragraph 74

Now, aware of the fact that 'a principle of arbitration' is required,⁷³ Edgeworth suggests the utilitarian apparatus as the best candidate to come to the rescue. Other possible candidates, he argues, such as distribution based on other conceptions of equity,⁷⁴ are less preferable solutions, and the utilitarian apparatus (used by the agents themselves) is the justified one:

Justice requires to be informed by some more definite principle, as Mill and Mr. Sidgwick reason well. The star of justice affords no certain guidance—for those who have loosed from the moorings of custom—unless it reflects the rays of a superior luminary—utilitarianism.⁷⁵

Now, it is a circumstance of momentous interest—visible to common sense when pointed out by mathematics—that one of the in general indefinitely numerous settlements between contractors is the utilitarian arrangement of the articles of contract, the contract tending to the greatest possible total utility of the contractors. In this direction, it may be conjectured, is to be sought the required principle.⁷⁶

The significant implication of this solution is that economic man is expected to demonstrate other motives apart from the maximization of his individual utility as a part of the process of exchange. In this, the ethical account of Henry Sidgwick (in which the agent is presented as motivated by a mixture of 'egoistic' and 'universal' hedonism i.e. utilitarianism⁷⁷) was actually penetrating the sphere of economic theory.

5) Changing the relations between economics and utilitarianism: Edgeworth and Marshall

Although the starting-point of Edgeworth's project of 1877-81 had not much to do with the field of economics, it nevertheless concluded with a monograph which reflected an original relation between economics and utilitarianism. As explained in the first section of this paper, British economists who accepted utilitarianism as an ethical guide tended to refer to it as a meta-criterion for assessing economic policy. From Edgeworth's point of view, and as exemplified by the way he edited *MS*, there was at the very least symmetry between the two fields: both are branches of the moral sciences, expressed with the same concepts and formulized with the language of

⁷³ Edgeworth (1881) p. 51

⁷⁴ Ibid.

⁷⁵ Ibid. p. 52

⁷⁶ Ibid. p. 53

⁷⁷ This point is elaborated in appendix IV in *MS*. Edgeworth (1881) pp. 102-104.

mathematics. From a later (20th century) point of view, one can interpret this in a more radical way as the inclusion of ethics within the economic discipline.⁷⁸

I would like to argue briefly in this concluding section that Edgeworth's new point of view was not an idiosyncratic caprice, but was shared, at least to some extent, with Alfred Marshall, and is indeed a part of a wider tendency that should be understood within the intellectual context of the time.

In reading Marshall's works in 1880,⁷⁹ (after Jevons recommended it as one of the latest contributions to mathematical economics⁸⁰), Edgeworth sent Marshall a copy of *N&O*, to which Marshall's responded:

I have now nearly read all the book you sent me and am extremely delighted by many things in it. There seems to be a very close agreement between us as to the promise of mathematics in the sciences that relate to man's actions. As to the interpretation of the Utilitarian dogma, I think you have made a great advance: but I have still a hankering after a mode of exposition in which the dynamical character of the problem is made more obvious.⁸¹

More striking than his reaction expressed in the letter is Marshall's note (probably written at the beginning of the 1880s⁸²) titled '*On Utilitarianism: A Summum Bonum*'. Within this note we find Marshall entertaining himself with mathematical representations of the utilitarian apparatus,⁸³ using and improving Edgeworth's basic integral (the same one as presented in 'HC' and therefore also in *MS*).⁸⁴ As mentioned by Whitaker, little trace of it remained in the *Principles*, but Marshall still proposed to use the ideas as late as 1912.⁸⁵ In between these two responses by Marshall to Edgeworth's 'exact utilitarianism', Marshall's review of *MS* appeared in *The Academy* (18 June 1881), focusing on Edgeworth's economic developments.⁸⁶

⁷⁸ See for example John Broome's essays: *Ethics out of Economics* (1999) and how Edgeworth's work could actually be interpreted as a pioneer work of this kind.

⁷⁹ *The Pure Theory of Foreign Trade and Domestic Values* (1879) and *Economics of Industry* (1879).

⁸⁰ Barbé p. 90 אולי להוסיף את המקום ממנו ברבה לקח את זה

⁸¹ Letter of 8 February, 1880

⁸² As assessed in Whitaker (1975) p. 317.

⁸³ While at the same time he reminds himself in a very Marshallian manner: "write as much as can be done without mathematics before beginning mathematics." Ibid. p. 317 (footnote 3).

⁸⁴ Ibid. p. 318-319.

⁸⁵ Ibid p. 317

⁸⁶ Marshall was very much supportive in his review: "This book shows clear signs of genius, and is a promise of great things to come". Nevertheless he expressed also his impression that the work is too mathematical and not enough clear and applicable. Whitaker (1975) pp. 266, 267; for further discussion of the two different approaches to economics see Creedy (1990) pp. 21-22.

The question of: 'how much of a utilitarian was Marshall anyway?' was broadly discussed in the literature.⁸⁷ In our current discussion it is only important to claim that although he was not as thoroughgoing utilitarian as was Edgeworth, Marshall had some tendencies to the doctrine, studying Bentham and Mill's philosophies in his youth, mixing it with some other philosophical influences,⁸⁸ and combining it with an evolutionary approach (in the spirit of Spencer). As opposed to Edgeworth, Marshall, the economist, was not interested in engaging himself directly with the philosophical disputes of his time, nor did he want to fully commit to any particular ethical school.⁸⁹ Famously, he also promoted the abolishment of Moral Sciences Tripos in which political economy was taught alongside moral philosophy at Cambridge.

Nevertheless, and regardless of the particular extent of his commitment to utilitarianism, Marshall did share with Edgeworth the aim of expressing utilitarian ideas by 'translating' them into the language of science. One way to do this was in the spirit of abstract ideas of evolution (i.e. social evolution as a utilitarian promoter),⁹⁰ the other, which is more relevant here, was Marshall's attempts to 'capture' utilities, that in his view *could not* be directly represented mathematically, by finding other representatives of them (such as market prices) that could actually be expressed mathematically. Marshall's development of the concept of 'consumer surplus' is an early example of this tendency.⁹¹

Marshall's own way of internalizing utilitarian reasoning into his economic theory (without accepting hedonism or using philosophical jargon), taken together with his positive reaction to Edgeworth's formalization of utilitarianism, suggests that the two indeed shared a similar new approach to the relation between economics and utilitarianism.

That new way of coping with utilitarianism will have some reactions in the future, both as inspiration to scholars and as an approach that drew fire in the form of some harsh criticism. But from the scholars point of view in the 1870s and the 1880s it

⁸⁷ See: Black (1990), Cready (1990) p. 23, Raffaelli (1996), Dardi (2010)

⁸⁸ Idealism and neo-Hegelianism in particular (through the interpretations of Maurice and Grote). See Cook (2009)

⁸⁹ See for example Marshall's comment in a meeting of the British Economic Association (1893): "Now it was true that some of the greatest economists had been utilitarians: but that was an accident. Their analysis was wholly independent of the utilitarian doctrine; it was, when rightly understood, common property to all ethical creeds". *Economic Journal* 3:11 p. 388

⁹⁰ Dardi (2010) pp. 418-422.

⁹¹ For further discussion of this point see Dardi (2010) pp. 411-417 Dardi nicely named this 'monetary utilitarianism' (p. 415)

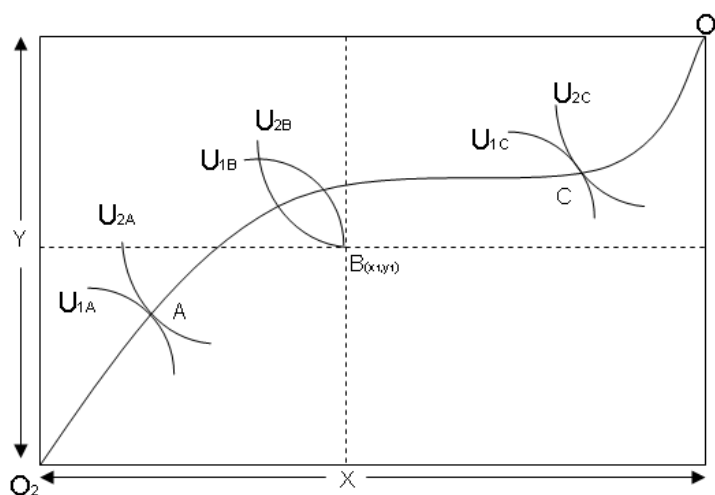
could be understood as a part of the 'spirit of the age'.⁹² In particular it was their mutual belief that one could ascribe more force to an idea by presenting it in a scientific and mathematic fashion.⁹³ It is not an accident that both scholars used the title 'new and old' to describe their intellectual challenges.⁹⁴ Within those *new* frameworks there was a place for expressing good old ideas in an up-to-date shape.

Appendix:

We can use the later pedagogical devise that captures Edgeworth's theoretical framework, "Edgeworth box", as an illustration to the need of utilitarianism as a Principle of Arbitration: The line that connects O₁ and O₂ is the contract line. Which mean that all the points on it reflect Jevons's equation:

$$\frac{du_1}{dx} = \frac{du_2}{dy},$$

The initial endowment is represented by the point B. any point inside the core (between the two indifferent curves U_{1B} and U_{2B}) and on the contract line are optional as a solution to the problem (and each of them resolved in different relative price). Therefore another principle is needed in order to decide between them.



⁹² See Winch's recent discussion of the debates between economists and their critics in the 1880's-90's under the title of "The old generation of political economists and the new". (see in particular the interpretation ascribed to the 'new' framework by Foxwell and Marshall. pp. 241-250) Winch (2009) chapter 9

⁹³ For the particular role of professionalization in Marshall's economics, and the difference between his approach to mathematics and Edgeworth's see Maloney (1985) pp. 181-185; Creedy (1990) pp. 21-22

⁹⁴ Edgeworth *N&O* of 1877 and Marshall's using of the title: "The old generation of economists and the new" in his address delivered in Cambridge (Oct 1896) see Marshall (1897).

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